education

Lefapha la Thuto la Bokone Bophirima
Noordwes Departement van Onderwys
North West Department of Education
NORTH WEST PROVINCE



NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

SEPTEMBER 2022

Stanmorephysics.com

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, 1 information sheet and an answer book of 20 pages.

Copyright reserved

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

downloaded from stanmorephysics.com

Copyright reserved

QUESTION 1

A study was conducted among eight (8) employees in a company to understand the relationship between the additional number of rest days given in a year and the productivity of each employee.

The results are shown in the table below.

Additional number of rest days	5	2	9	1	3	10	4	6
Productivity of the employee (in %)	85	63	90	56	70	88	72	62

1.1 Calculate the correlation coefficient of the data. (1)

1.2 Consider the correlation coefficient calculated in QUESTION 1.1 and the four phrases below. Only write down the letter of the phrase that best describes the relationship between the variables in the given data.

A. Fairly strong, negative correlation

B. Very weak, positive correlation

C. Fairly strong, positive correlation

D. Perfect, positive correlation (1)

1.3 Determine the equation of the least squares regression line of the data. (3)

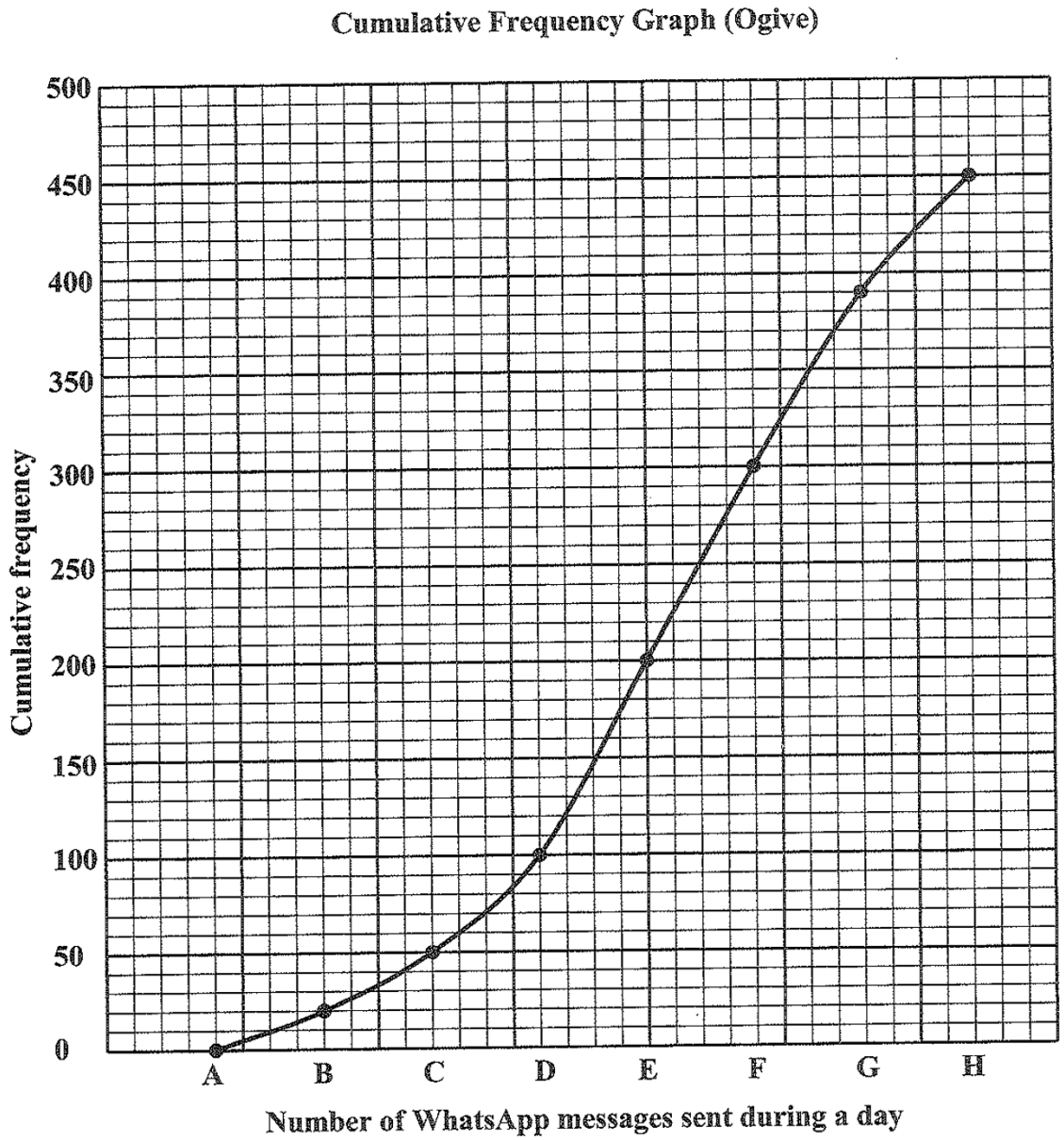
1.4 Use the equation obtained in QUESTION 1.3 to predict an employee's productivity if he/she was given eight (8) additional rest days. (2)

1.5 Should the regression line in QUESTION 1.3 be used to predict the productivity of an employee if he/she was given thirty (30) additional rest days in the year? Explain your answer. (2)
[9]

downloaded from stanmorephysics.com

QUESTION 2

A survey was conducted among a number of learners who were using the same cellphone company to establish the number of WhatsApp messages that they sent during a day. The results were summarised in the partially completed cumulative frequency graph (ogive) below.



Copyright reserved

The data shown in the cumulative frequency graph is also summarised in the partially completed frequency table below.

WhatsApp messages sent	Frequency
$40 \leq x < 80$	20
$80 \leq x < 120$	30
$120 \leq x < 160$	Q
$160 \leq x < 200$	R
$200 \leq x < 240$	100
$240 \leq x < 280$	90
$280 \leq x < 320$	60

2.1 Write down the value of A, a tick mark on the horizontal axis of the cumulative frequency curve. (1)

2.2 How many learners participated in this survey? (1)

2.3 Calculate the values of Q and R, the frequencies in the frequency table above. (3)

2.4 Estimate the mean number of WhatsApp messages sent per day. (2)

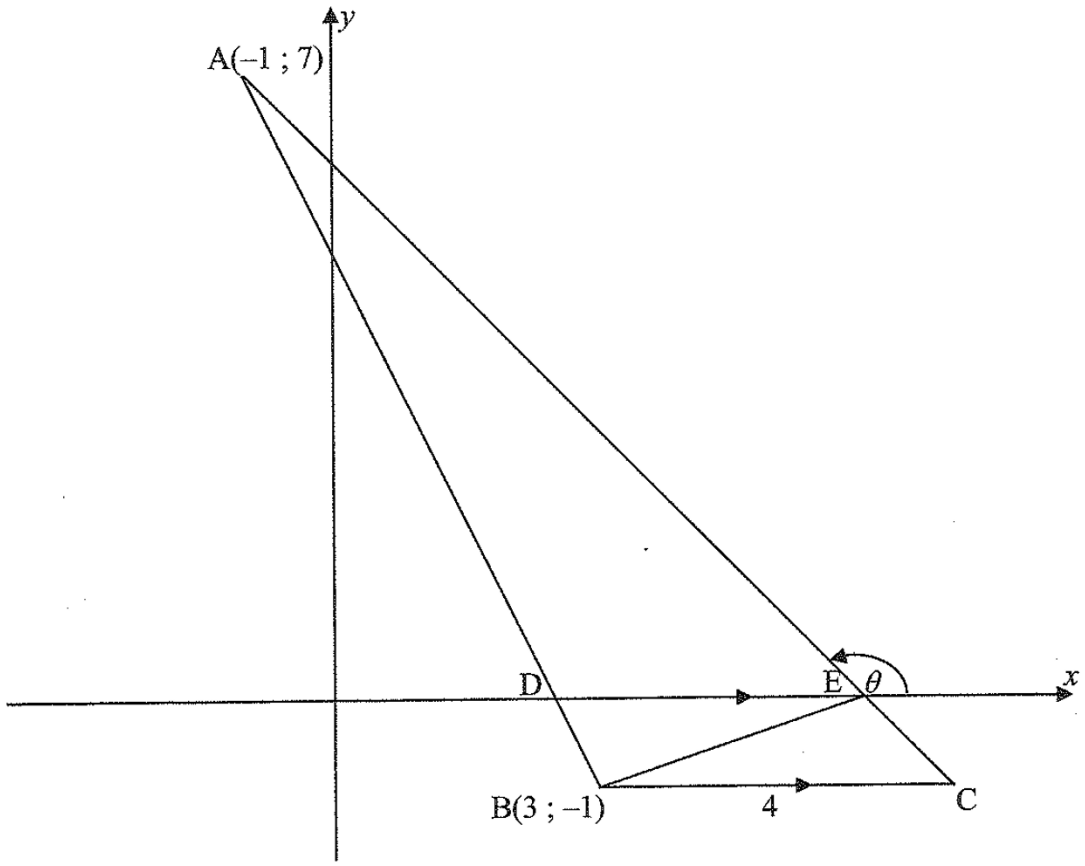
2.5 Calculate the interquartile range. (3)

2.6 The cellphone company wants to introduce a new rule where you are not allowed to send more than 200 WhatsApp messages per day. If this rule is applied to this group of learners on the existing data, how would the standard deviation for the data be affected? Motivate your answer. (2)
[12]

Stanmorephysics.com

QUESTION 3

In the diagram below, A(–1 ; 7), B(3 ; –1) and C are the vertices of Δ ABC. The angle of inclination of AC is θ. BC || x-axis and BC = 4 units. D and E are the x-intercepts of the lines AB and AC respectively. B and E are joined.



3.1 Write down the equation of the line BC. (1)

3.2 Write down the x-coordinate of C. (1)

3.3 Determine the equation of the line AC. (4)

3.4 Calculate the size of $A\hat{C}B$. (4)

3.5 If point K is the reflection of E in the line BC:

3.5.1 Write down the coordinates of K. (3)

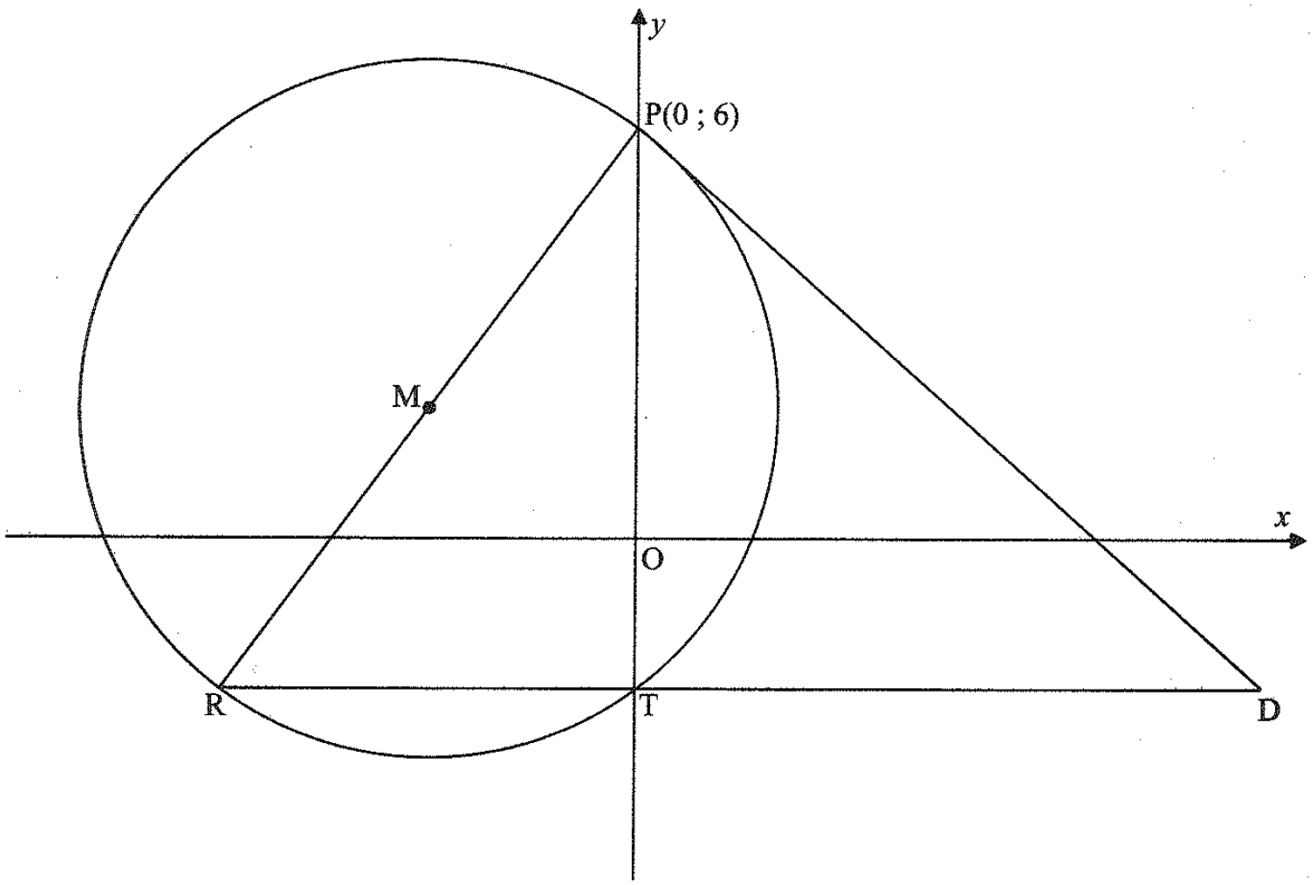
3.5.2 Calculate the area of quadrilateral BECK. (3)

3.6 F is a point in the first quadrant, such that it forms an equilateral Δ BFC. Calculate the coordinates of F. (5)
[21]

Copyright reserved

QUESTION 4

In the diagram below, a circle with centre M and a tangent PD to the circle at P(0 ; 6) are drawn. The equation of the circle is given as $x^2 + 6x + y^2 - 4y - 12 = 0$.
PR is a diameter, T is a y-intercept and chord RT is produced to D. RTD $\parallel$ x-axis.



4.1 Determine the coordinates of M. (3)

4.2 Determine the length of PR. (2)

4.3 Determine the equation of the tangent PD. (3)

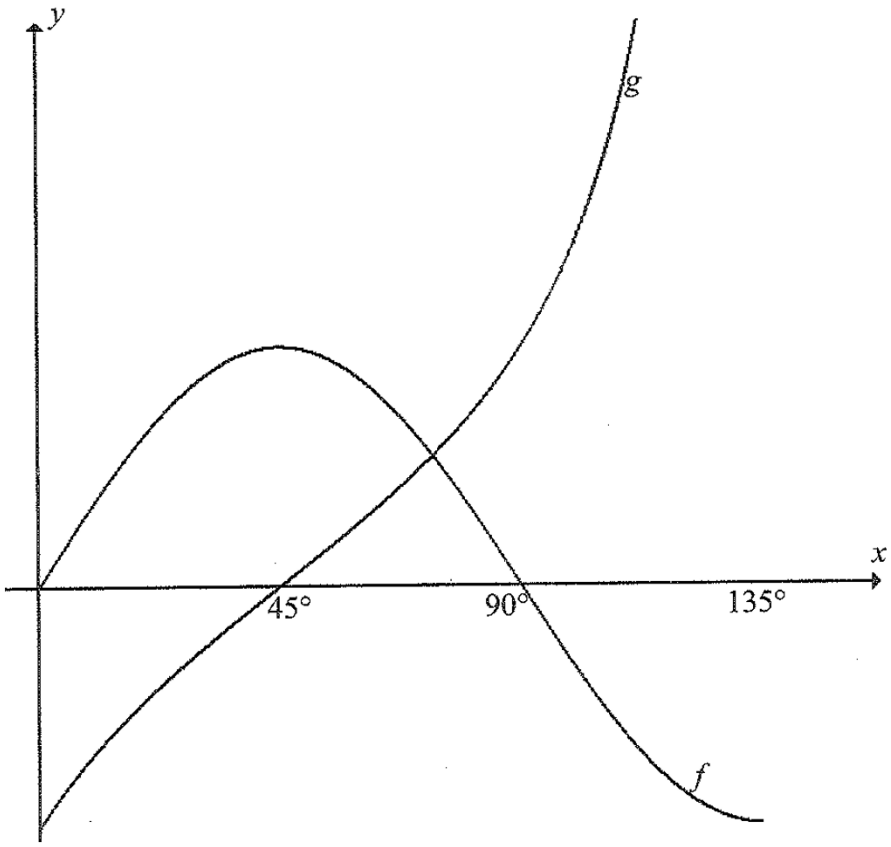
4.4 Determine the coordinates of D. (4)

4.5 The circle, having centre M, is translated to the right such that the point D lies on the translated circle. Determine the coordinates of the possible centres of the translated circle. (4)
[16]

Copyright reserved

QUESTION 5

In the diagram below, $f(x) = \sin ax$ and an incompleted $g(x) = \tan(x + b)$ are drawn for the interval $x \in [0° ; 135°]$.



5.1 Write down the values of a and b. (2)

5.2 Write down the range of f. (2)

5.3 Write down the period of g. (1)

5.4 Write down the equation of the asymptote of g for the given interval. (1)

5.5 Write down TWO values of x, in the given interval, where $f(x) = 1 - g(x)$ (2)

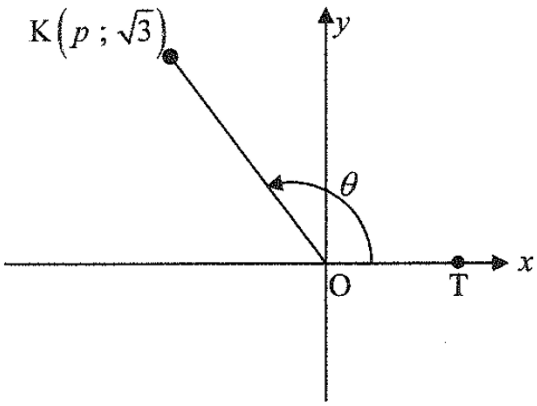
5.6 For which value(s) of x, in the given interval, is $f'(x).g'(x) > 0$? (2)

[10]

Copyright reserved

QUESTION 6

6.1 In the diagram, $K(p\ ;\ \sqrt{3})$ is a point in the 2nd quadrant. T is a point on the positive x-axis and obtuse $K\hat{O}T = \theta$.



6.1.1 Write down the value of $\tan\theta$ in terms of p. (1)

6.1.2 If $\theta = 120°$, **WITHOUT using a calculator**, calculate the value of p. (3)

6.2 If $\cos 42° = t$, **WITHOUT using a calculator**, write the following expressions in terms of t:

6.2.1 $\sin 48°$ (2)

6.2.2 $\cos 84°$ (3)

6.2.3 $\cos 72°$ (5)



6.3 Simplify the following expression to ONE trigonometric ratio:

$$\frac{\sin x}{\sin 2x} + \frac{\sin(180° - x)}{\cos x} - \frac{\sin^2 x + \cos^2 x}{2\sin(90° + x)}$$ (6)

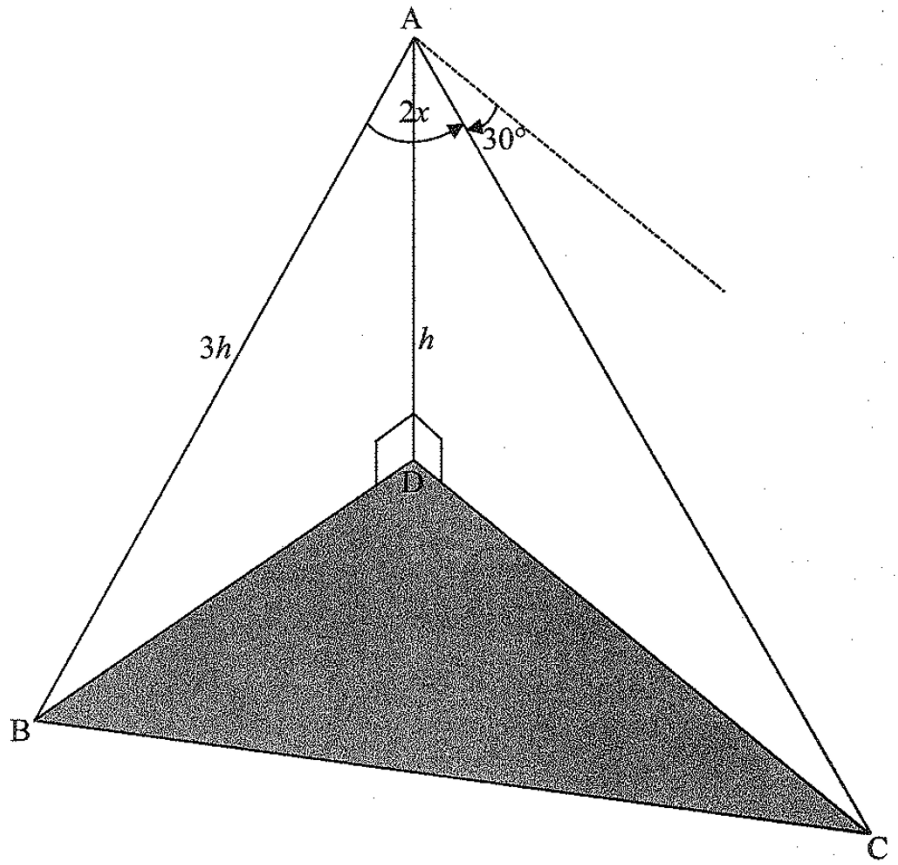
6.4 Consider $f(x) = \sin(2\theta - 15°).\cos(\theta - 30°) + \cos(2\theta - 15°).\sin(\theta - 30°)$
Determine the general solution of $f(x) = 0{,}8$ (7)

6.5 It is given that $4^{\sin x} = 6$, $6^{\cos x} = 8$ and $8 = 32^{1 - 2\sin^2 x}$
WITHOUT solving for x, determine the value of $\tan 2x$. (5)
[32]

Copyright reserved

QUESTION 7

In the diagram below, AD is a vertical pole having height h metres. B, D and C are three points in the same horizontal plane. AB and AC are cables and the angle of depression from A to C is 30°. AB = $3h$ and $\hat{BAC} = 2x$.



7.1 Write down the size of $\hat{ACD}$. (1)

7.2 Determine the distance AC in terms of h. (2)

7.3 Calculate the size of $\hat{ABD}$. (2)

7.4 Calculate the size of x if BC = $\sqrt{7}h$. (5)

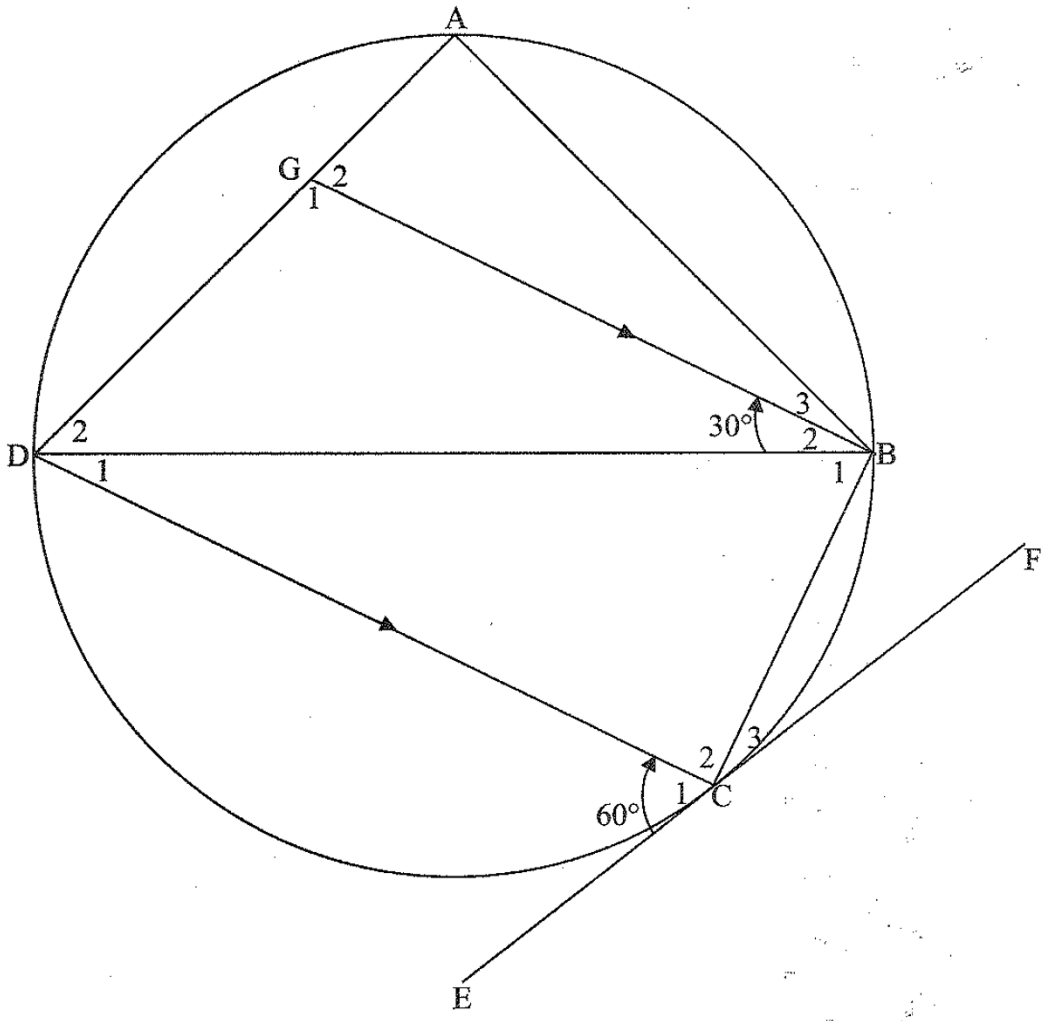
[10]

Copyright reserved

Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

In the diagram, ABCD is a cyclic quadrilateral. G is a point on AD such that BG || CD. ECF is a tangent to the circle at C. BD is a chord of the circle.
$G\hat{B}D = 30°$ and $D\hat{C}E = 60°$.



8.1 Calculate, with reasons, the size of:

8.1.1 $\hat{D}_1$ (1)

8.1.2 $\hat{B}_1$ (2)

8.1.3 $\hat{C}_2$ (1)

8.1.4 $D\hat{A}B$ (2)

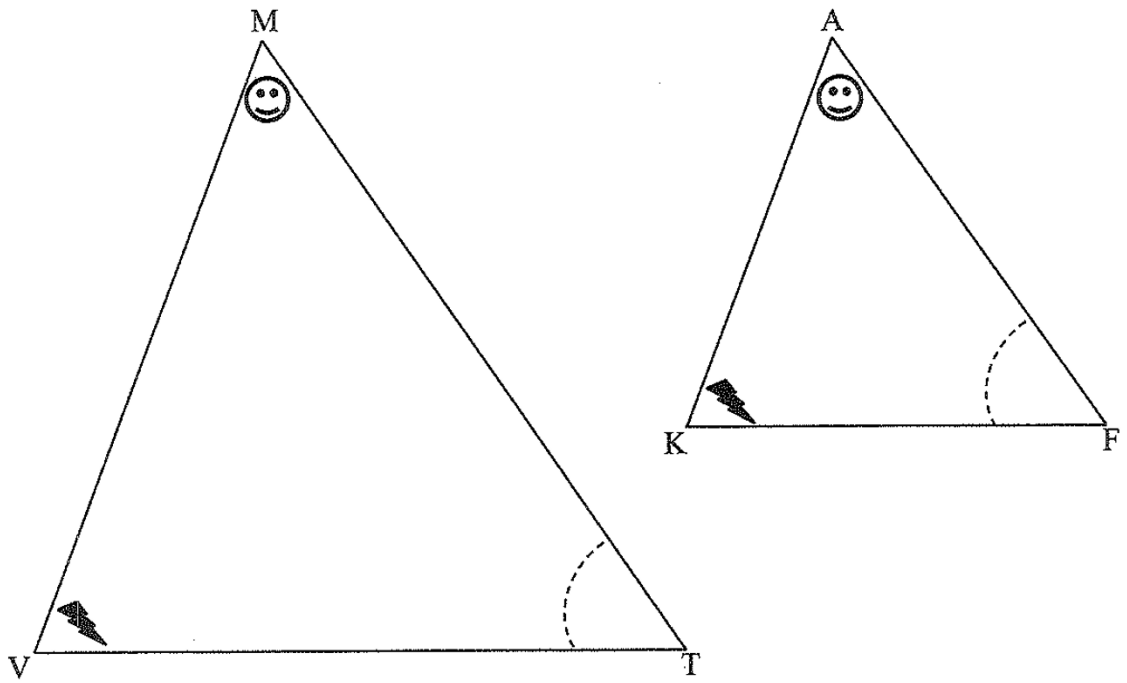
8.2 Is BD a diameter of the circle? Motivate your answer. (2)
[8]

Stanmorephysics.com

Copyright reserved

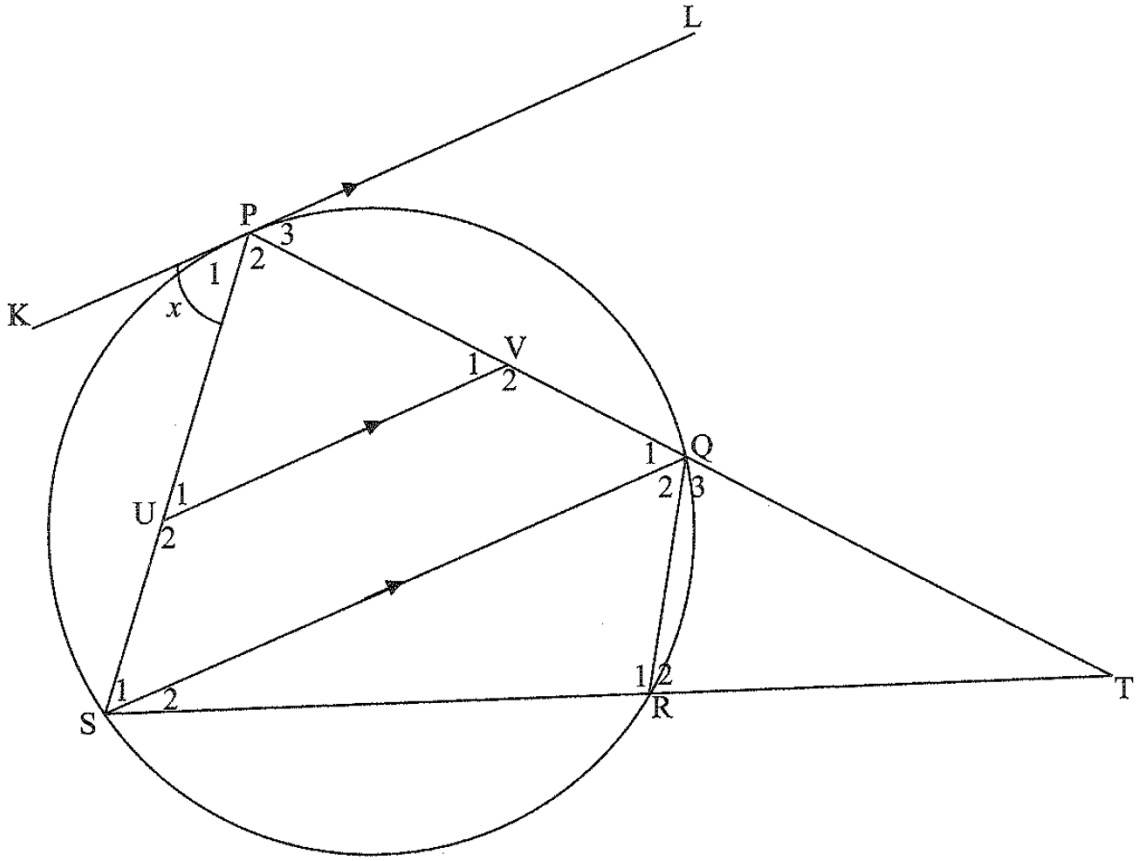
QUESTION 9

9.1 In the diagram below, ΔMVT and ΔAKF are drawn such that $\hat{M}=\hat{A}$, $\hat{V}=\hat{K}$ and $\hat{T}=\hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{MV}{AK}=\frac{MT}{AF}$ (7)

9.2 In the diagram below, cyclic quadrilateral PQRS is drawn. Chord SR produced and chord PQ produced meet at T. KPL is a tangent to the circle at P. U and V are points on PS and PQ respectively such that UV || SQ || KL. Let $K\hat{P}S = x$



9.2.1 Write down, with reasons, FIVE other angles EACH equal to x. (5)

9.2.2 Prove that:

(a) PQ = PS (1)

(b) KP is a tangent to the circle that passes through the points P, U and V, at P. (1)

9.2.3 Give a reason why $\frac{PU}{PS} = \frac{PV}{PQ}$ (1)

9.2.4 Prove that:

(a) $\Delta PTS \,|||\, \Delta RTQ$ (4)

(b) PT.RQ = PS.RT (2)

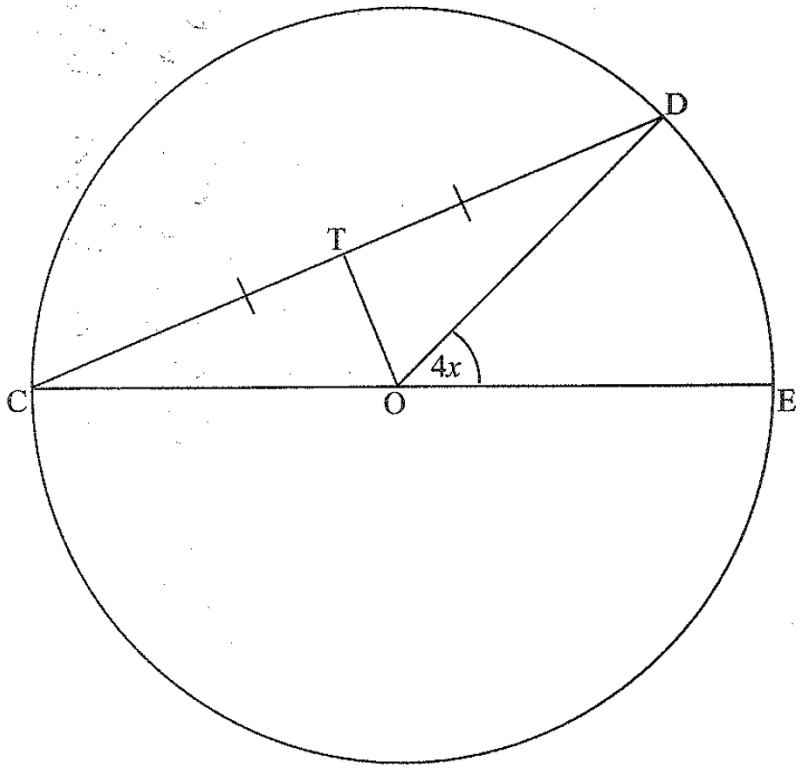
(c) $PQ = \sqrt{PS.PT - TS.RQ}$ (4)

[25]

Copyright reserved

QUESTION 10

In the diagram below, the circle with centre O, passes through the points C, D, and E.
OT bisects chord CD. $D\hat{O}E = 4x$



Prove, with reasons, that $\sin x.\cos x = \frac{OT}{CE}$ **[7]**

TOTAL: 150

Copyright reserved

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i} \qquad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad \text{P(A or B)} = \text{P(A)} + \text{P(B)} - \text{P(A and B)}$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

Copyright reserved