education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

# NATIONAL SENIOR CERTIFICATE

## GRADE 12

## MATHEMATICS PAPER 1

## SEPTEMBER 2023

**MARKS: 150**

**TIME: 3 hours**

Stanmorephysics

**This question paper consists of 10 pages and a one-page information sheet**

## QUESTION 1

1.1 Solve for $x$

1.1.1 $(3-x)(2-x)=0$ (2)

1.1.2 $2x^2+7x=2$ (Correct to 2 decimal places) (4)

1.1.3 $4+5x\geq 6x^2$ (4)

1.1.4 $9^x+9=10.3^x$ (4)

1.2 Solve for $x$ and $y$:

$y-2x=-1$

$y^2-3xy=-2$ (6)

1.3 If $\frac{x}{y}+\frac{y}{x}=\frac{17}{4}$, calculate two values of $\frac{x}{y}$. (5)

**[25]**

## QUESTION 2

2.1 Consider the following quadratic sequence:

319 ; 280 ; 243 ; 208 ; 175 ; 144 ; … .

2.1.1 Show that the n$^{th}$ term of the sequence is $T_n=n^2-42n+360$. (4)

2.1.2 Determine which terms has a value of 0? (3)

2.1.3 Which term in the sequence will have the lowest value? (2)

2.2 $3t$ ; $4t-1$ ; 23 are the first three terms of an arithmetic sequence.

2.2.1 Prove that $t=5$. (2)

2.2.2 Calculate the sum of the first 50 terms of the series. (4)

**[15]**

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## QUESTION 3

3.1 The following geometric series is given:

$$4+12+36+\ldots \text{to 15 terms}$$

3.1.1 Write the series in sigma notation. (2)

3.1.2 Calculate the series. (2)

3.1.3 Is this series convergent? Provide a reason for your answer. (2)

3.2 The sum of an infinite, geometric series is $13{,}5\,(r \neq 1)$.

The sum of the same series, calculated from the third term is 1,5.

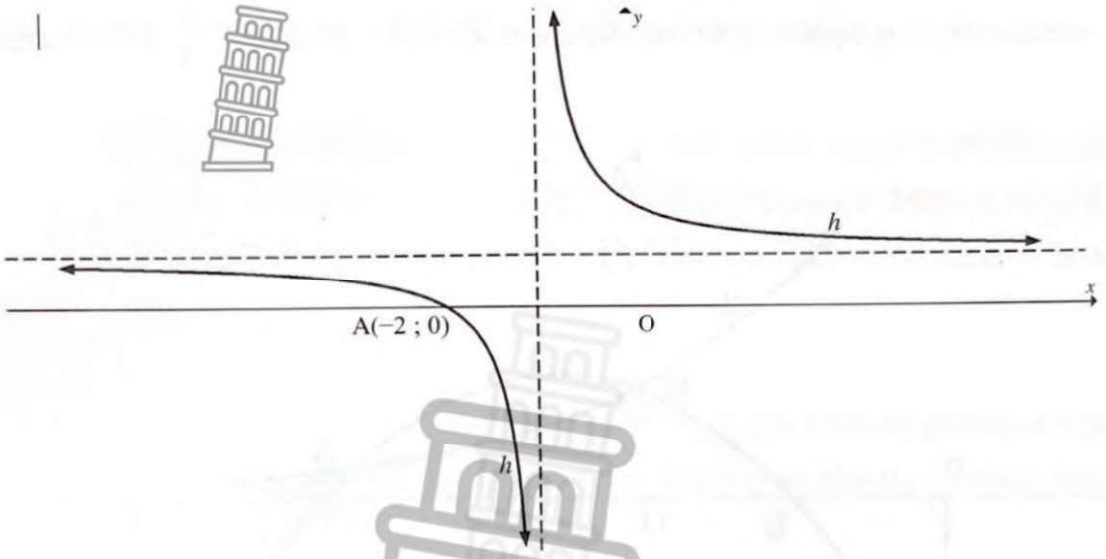
3.2.1 Calculate $r$ if $r > 0$. (4)

3.2.2 Hence, determine the first THREE terms of the series. (2)

**[12]**

## QUESTION 4

In the diagram, $A(-2\,;0)$ is the $x$-intercept of $h(x)=\dfrac{a}{x+1}+2$.




4.1 Calculate $a$. (2)

4.2 Calculate the average gradient of $h$ between $x=-4$ and $x=-2$. (3)

4.3 For which values of $x$ is $h(x)>4$? (2)

4.4 Determine the equation of the axis of symmetry of $h$ that has a positive gradient. Leave your answer in the form $y=\ldots$ (2)

4.5 Write down the equations of the asymptotes of $g$, if $g$ is the reflection of $h$ over the $x$-axis. (2)

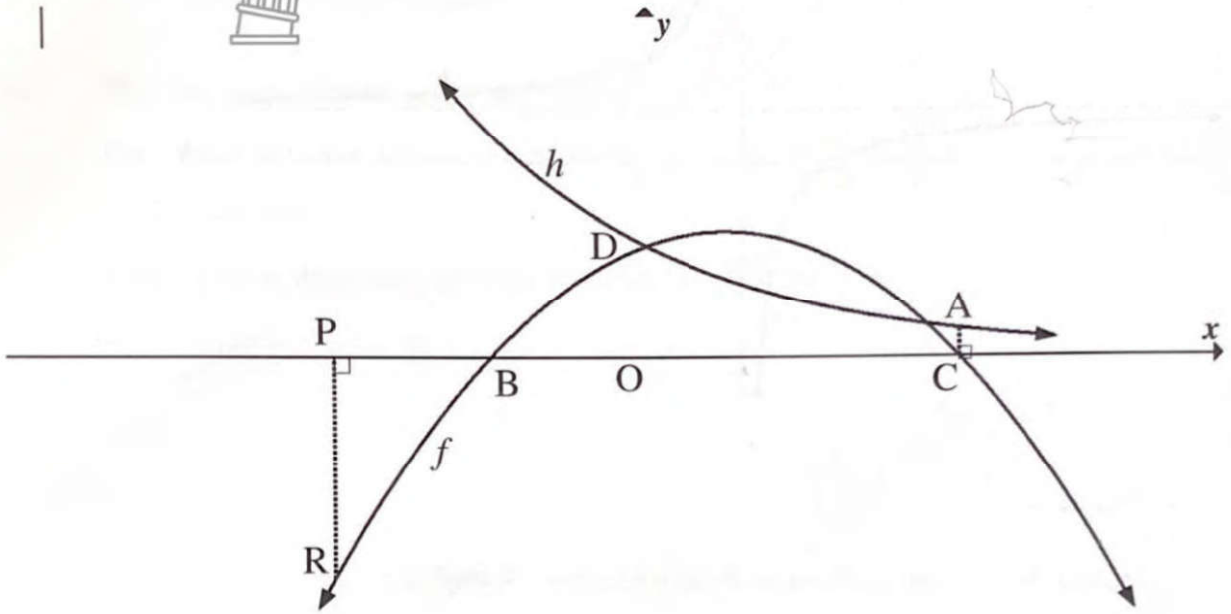
**[11]**

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## QUESTION 5

Sketched below is $f(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + k$ and $h(x) = q^x$. The $x$-intercepts of $f$ are B and C. $h$ and $f$ intersect the $y$-axis at D. A is a point on $h$ vertically above C with a $y$-value of $\frac{1}{4}$. PR is perpendicular to the $x$-axis.




5.1 Calculate the coordinates of D. (1)

5.2 Write down the value of $k$. (1)

5.3 Calculate the length of BC. (4)

5.4 Calculate the length of PR if OP is 2 units. (2)

5.5 Consider $h(x) = q^x$

5.5.1 Calculate the value of $q$. (3)

5.5.2 Hence, determine the equation of $h^{-1}$, in the form $y = \ldots$ (2)

5.6 Write down the range of $h$. (1)

5.7 Determine the equation of $g$, the tangent to $f$ at B. (5)

5.8 For which values of $x$ is $x.f'(x) < 0$? (2)

**[21]**

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## QUESTION 6

6.1 Jane deposits R$x$ into an investment account. How long will it take for the value of the investment to double if the interest rate is 5,4% p.a. compounded annually? (3)

6.2 Thabo starts a printing company and needs to borrow money for start-up costs. He can make equal monthly payments of R3 300. What amount can Thabo borrow if the interest rate on the loan is 12% p.a. compounded monthly and the loan is granted over 5 years? (4)

6.3 A group of investors consider investing in a fund that promises growth at a rate of 5% p.a. compounded quarterly. Calculate the effective annual percentage rate of the growth promised. (3)

6.4 Sarah is 18 years old and wishes to accumulate R10 000 000 by the month before her 50$^{th}$ birthday. She will deposit equal monthly payments into an account that pays 15% p.a. compounded monthly. The first payment starts on her 18$^{th}$ birthday and the last payment one month before her 50$^{th}$ birthday.
Calculate the monthly instalment that Sarah will make. (3)

**[13]**

## QUESTION 7

7.1 Given: $f(x) = -2x^2 + 1$
Determine $f'(x)$ from first principles. (5)

7.2 Determine:

7.2.1 $f'(x)$ if $f(x) = \frac{1}{2}x^2 - \frac{5}{x}$ (3)

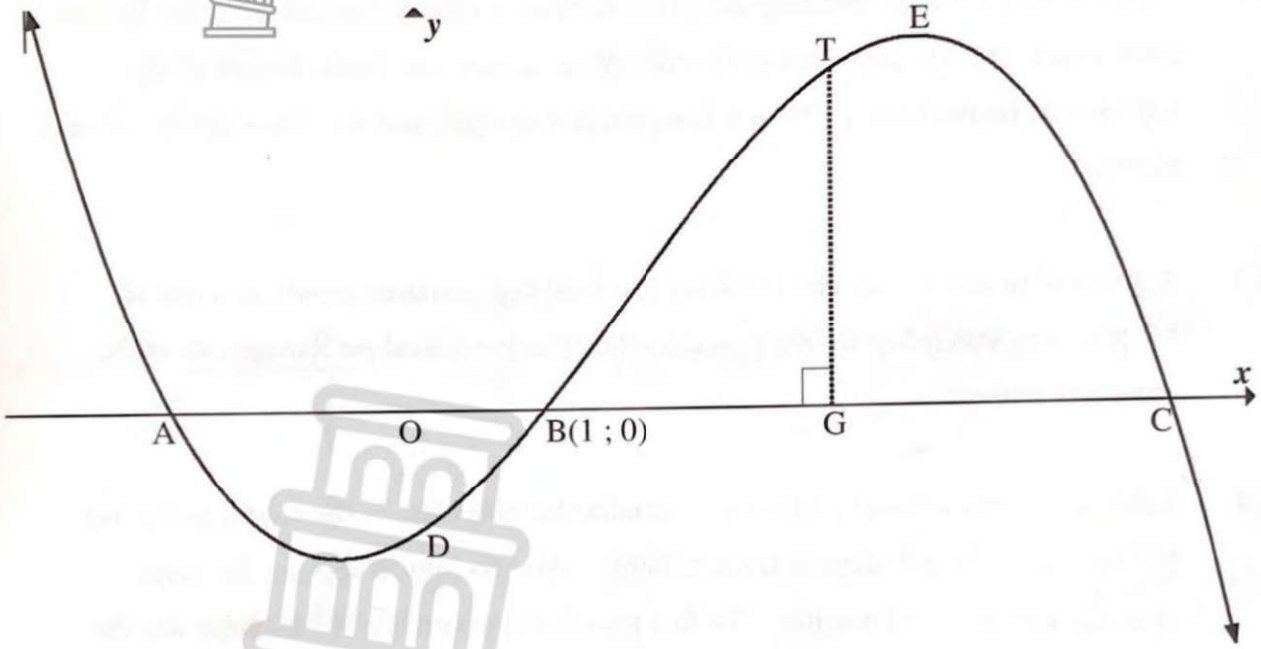
7.2.2 $D_x\left[\frac{-2x^2 + \sqrt[4]{x}}{x^2}\right]$ (3)

**[11]**

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## QUESTION 8

In the diagram, the graph of $f(x) = -x^3 + 5x^2 + 8x - 12$ is drawn. A, B and C are the $x$-intercepts of $f$. E is a turning point of $f$. T is a point on $f$ and G is a point on the $x$-axis such that TG is perpendicular to the $x$-axis. D is the $y$-intercept of $f$.




8.1 Calculate the coordinates of C if B(1 ; 0). (4)

8.2 Determine the coordinates of E. (5)

8.3 For which values of $x$ will $f$ be concave up? (2)

8.4 Calculate the length of OG if the tangent to the curve at T is parallel to the tangent to the curve at D. (5)

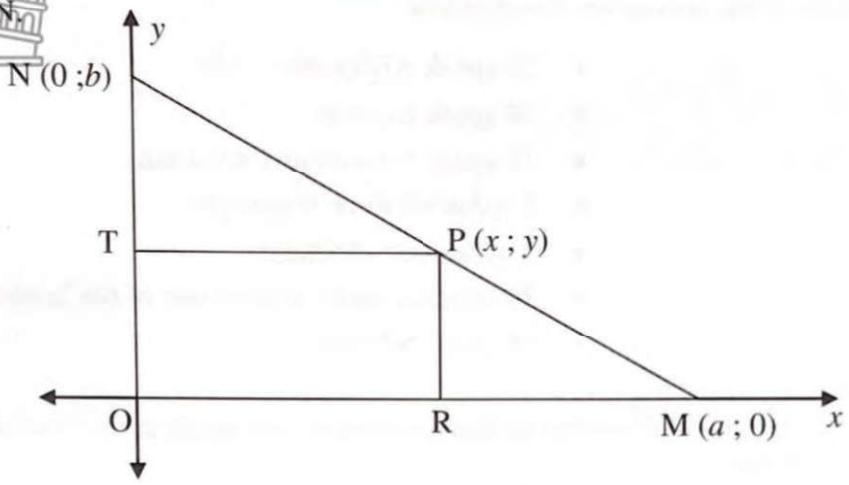
8.5 Determine the value of $m$ if $y = mx + c$ intersects $f$ perpendicularly at $x = 5$. (3)

**[19]**

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## QUESTION 9

A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the figure below. He allocates a rectangular piece of land PTOR to his daughter, giving her the freedom to choose P anywhere along the boundary MN. Let $OM = a$, $ON = b$ and $P(x\,;\,y)$ be any point on MN.




9.1 Determine an equation for MN in terms of $a$ and $b$. (2)

9.2 Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN. (6)

**[8]**

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## QUESTION 10

10.1 A language survey was done with 45 grade 12 learners in Mbombela Secondary School.

The aim of the study was to find out what language learners speak in their homes.

The results of the survey are listed below:

- 22 speak Afrikaans
- 24 speak English
- 11 speak SiSwati and Afrikaans
- 5 speak all three languages
- 3 speak only Afrikaans
- 38 learners speak at least one of the languages at home
- 18 speak SiSwati

10.1.1 Determine the number of learners that do not speak any of the named languages at home. (1)

10.1.2 If $x$ learners speak English and SiSwati but not Afrikaans, draw a Venn diagram to represent the information. (4)

10.1.3 Solve for $x$. (2)

10.1.4 What is the probability that a learner speaks only two of the three languages at home? (2)

10.2 Three brothers Owen, Richard and Robert are to run a race which has 8 runners in total.

The eight competitors line up one to a lane in the lanes numbered 1 to 8.

10.2.1 Write down the total number of possible arrangements of the runners at the starting line. (1)

10.2.2 Calculate the total number of arrangements in which the three brothers are all next to each other. (3)

10.2.3 Calculate the probability that Owen is in lane 1, Robert is in lane 2 and Richard is in lane 3. (2)

**[15]**

**TOTAL: 150**

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