education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PAPER 1

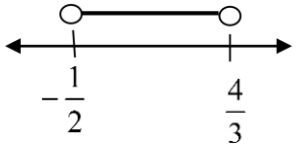
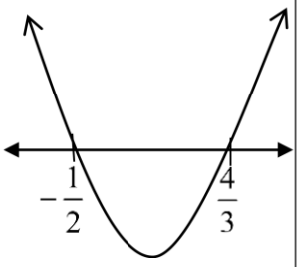
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MARKING GUIDELINES

Stanmorephysics

MARKS: 150

This marking guideline consists of 13 pages.

QUESTION 1 / VRAAG 1		
1.1.1	$(3-x)(2-x)=0$ $x=3 \quad or \quad x=2$	✓ ✓answers (2)
1.1.2	$2x^2+7x=2$ $2x^2+7x-2=0$ $x=\dfrac{-(7)\pm\sqrt{7^2-4(2)(-2)}}{2(2)}$ $x=\dfrac{-7\pm\sqrt{65}}{4}$ $x=0,27 \quad or \quad x=-3,77$ Penalise 1 mark for incorrect rounding	✓ standard form ✓ substitution into correct formula ✓ ✓ answers (4)
1.1.3	$4+5x>6x^2$ $6x^2-5x-4<0$ $(3x-4)(2x+1)<0$  **OR**  $-\dfrac{1}{2}<x<\dfrac{4}{3}$ OR $x\in\left(-\dfrac{1}{2};\dfrac{4}{3}\right)$	✓ factors ✓ critical values ✓✓ answer (4)
1.1.4	$9^x+9=10.3^x$ $3^{2x}-10.3^x+9=0$ $(3^x-9)(3^x-1)=0$ $3^x=3^2 \quad or \quad 3^x=3^0$ $x=2 \quad or \quad x=0$	✓ 3^{2x} ✓ factors ✓exponential law ✓ answer (4)

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1.2	$y-2x=-1$ $y=2x-1\ldots.(1)$ $y^2-3xy=-2\ldots.(2)$ $(2x-1)^2-3x(2x-1)=-2$ $4x^2-4x+1-6x^2+3x=-2$ $-2x^2-x+3=0$ $2x^2+x-3=0$ $(2x+3)(x-1)=0$ $x=-\frac{3}{2}\ \ or\ \ x=1$ $y=2\left(-\frac{3}{2}\right)-1\ or\ \ y=2(1)-1$ $y=-4\ \ or\ \ y=1$ **OR** $y-2x=-1$ $x=\frac{y+1}{2}\ldots.(1)$ $y^2-3xy=-2\ldots.(2)$ $y^2-3\left(\frac{y+1}{2}\right)y=-2$ $2y^2-3y^2-3y+4=0$ $-y^2-3y+4=0$ $y^2+3y-4=0$ $(y+4)(y-1)=0$ $y=-4\ \ or\ \ y=1$ $x=-\frac{3}{2}\ \ or\ \ x=1$	✓ equation 1 ✓ substitution ✓ standard form ✓ factors ✓ x-values ✓ y-values (6) ✓ equation 1 ✓ substitution ✓ standard form ✓ factors ✓ y-values ✓ x-values (6)
1.3	Multiply every term by LCD $4xy$ $4x^2+4y^2=17xy$ $4x^2-17xy+4y^2=0$ $(4x-y)(x-4y)=0$ $4x-y=0$ $\frac{x}{y}=\frac{1}{4}$ $x-4y=0$ $\frac{x}{y}=4$	✓ multiply by LCD ✓ standard form ✓ factors ✓ answer ✓ answer (5)

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	OR $\frac{x}{y}+\frac{y}{x}=\frac{17}{4}$ $\frac{x}{y}+\frac{1}{\frac{x}{y}}=\frac{17}{4}$	✓ fraction
	$\frac{x^2}{y^2}-\frac{17x}{4y}+1=0$	✓ standard form
	$\frac{x}{y}=\frac{\frac{17}{4}\pm\sqrt{\left(\frac{17}{4}\right)^2-4(1)(1)}}{2(1)}$	✓ substitution into correct formula
	$=\frac{\frac{17}{4}\pm\sqrt{\frac{225}{16}}}{2}$	✓ simplification
	$\frac{x}{y}=4 \text{ or } \frac{x}{y}=\frac{1}{4}$	✓ answers (5)
		[25]

QUESTION 2 / VRAAG 2

2.1	319 ; 280 ; 243 ; 208 ; 175 ; 144 ; ….	
2.1.1	319 280 243 208 −39 −37 −35 2 2 $2a=2 \quad 3a+b=-39 \quad a+b+c=319$ $a=1 \quad 3(1)+b=-39 \quad 1-42+c=319$ $b=-42 \quad c=360$ $T_n=n^2-42n+360$	✓ First differences ✓ $2a=2$ ✓ $3(1)+b=-39$

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		✓ $1 - 42 + c = 319$ (4)
2.1.2	$n^2 - 42n + 360 = 0$ $(n-12)(n-30) = 0$ $n = 12 \quad or \quad n = 30$	✓ $T_n = 0$ ✓ factors ✓ answer (3)
2.1.3	$2n - 42 = 0$ **OR** $n = \frac{-(-42)}{2(1)}$ $n = 21$ $n = 21$ T_{21} is lowest	✓ derivative = 0 ✓ answer (2) OR ✓ substitution into $x = \frac{-b}{2a}$ ✓ answer (2)
2.2	$3t\ ;\ 4t-1\ ;\ 23$	
2.2.1	$3t\ ;\ 4t-1\ ;\ 23$ $4t-1-3t = 23-(4t-1)$ $4t-1-3t = 23-4t+1$ $5t = 25$ $t = 5$	✓ subtraction ✓ simplification (2)
2.2.2	$15\ ;\ 19\ ;\ 23$ $a = 15 \quad d = 4$ $S_{50} = \frac{50}{2}[2(15) + (49)4]$ $= 5650$	✓ $a = 15$ ✓ value of d ✓ substitution into correct formula ✓ answer (4)
		[15]

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QUESTION 3

3.1	$x = 4 + 12 + 36 + \ldots$to 15 terms	
3.1.1	$T_n = a.r^{n-1}$ $T_n = 4.3^{n-1}$ $\sum_{n=1}^{15}\left(4.3^{n-1}\right)$	✓ $T_n = 4.3^{n-1}$ ✓ answer (2)
3.1.2	$S_n = \frac{a\left(r^n - 1\right)}{r-1}$ $S_n = \frac{4\left(3^{15} - 1\right)}{3-1}$ $= 28\,697\,812$	✓ substitution into the correct formula ✓ answer (2)
3.1.3	$r = 3$ and not between -1 and 1 $\therefore$ not converging	✓ $r = 3$ not in $-1 < r < 1$ ✓ answer (2)
3.2.1	$S_\infty = \frac{a}{1-r} = 13{,}5$ $a = 13{,}5(1-r)$ $S_\infty = \frac{ar^2}{1-r} = 1{,}5$ $ar^2 = 1{,}5(1-r)$ $a = \frac{1{,}5(1-r)}{r^2}$ $13{,}5(1-r) = \frac{1{,}5(1-r)}{r^2}$ $r^2 = \frac{1{,}5}{13{,}5}$ $= \frac{1}{9}$ $r = \pm\frac{1}{3}$ But $r > 0$ $\therefore r = \frac{1}{3}$	✓ substitution into correct formula ✓ $a = 13{,}5(1-r)$ ✓ equating ✓ $r = \frac{1}{3}$ (4)

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3.2.2	$\frac{a}{1-\frac{1}{3}}=13,5$ $a=9$ $9+3+1+\ldots$	✓ value of a ✓ answer (2)
		[12]

QUESTION 4

4.1	$0=\frac{a}{-2+1}+2$ $a=2$	✓substitution ✓ answer (2)
4.2	Average gradient $=\frac{h(-2)-h(-4)}{-2-(-4)}$ $=\frac{\frac{2}{-2+1}+2-\left(\frac{2}{-4+1}+2\right)}{2}$ $=-\frac{2}{3}$	✓ $h(-4)$ and $h(-2)$ ✓ substitution ✓ answer (3)
4.3	$-1<x<0$	✓critical values ✓answers (2)
4.4	$y=x+1+2$ $y=x+3$ **OR** $y=x+c$ $2=-1+c$ $c=3$ $y=x+3$	✓ gradient ✓ answer (2) **OR** ✓ value of c ✓ answer (2)
4.5	$g(x)=\frac{-2}{x+1}-2$ $x=-1$ $y=-2$	✓ horizontal asymptote ✓ vertical asymptotes (2)
		[11]

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QUESTION 5		
	$h(x)=q^x \quad f(x)=-\frac{1}{2}x^2+\frac{1}{2}x+k \quad g(x)=mx+c$	
5.1	$h(x)=q^0$ $=1 \qquad D(0\,;1)$	✓ answer (1)
5.2	$k=1$	✓ answer (1)
5.3	$f(x)=-\frac{1}{2}x^2+\frac{1}{2}x+1$ $x^2-x-2=0$ $(x-2)(x+1)=0$ $x=2$ or $x=-1$ BC = 3units	✓ $f(x)=0$ ✓ factors ✓ both x values ✓ answer (4)
5.4	$f(-2)=-\frac{1}{2}(-2)^2+\frac{1}{2}(-2)+1$ $=-2$ PR = 2 units	✓ substitution of -2 ✓ answer (2)
5.5.1	$h(x)=q^x$ $\frac{1}{4}=q^2$ $q=\frac{1}{2}$	✓ A$\left(2;\frac{1}{4}\right)$ ✓ substitution of $\left(2;\frac{1}{4}\right)$ ✓ value of q (3)
5.5.2	$h(x)=\left(\frac{1}{2}\right)^x$ $h^{-1}(x)=\log_{\frac{1}{2}}x$	✓ equation of h ✓ equation of h^{-1} (2)
5.6	$y>0$	✓ answer (1)
5.7	$f'(x)=-x+\frac{1}{2}$ $f'(-1)=-(-1)+\frac{1}{2}=\frac{3}{2}$ $y-y_1=m(x-x_1)$ B(–1 ; 0) $y-0=\frac{3}{2}(x-(-1))$ $y=\frac{3}{2}x+\frac{3}{2}$	✓ derivative ✓ derivative $x=-1$ ✓ $f'(-1)=\frac{3}{2}$ ✓ substitution of B ✓ tangent (5)
5.8	$x<-1$ *or* $x>2$ OR $x\in(-\infty\,;1)\cup(2;\infty)$	✓✓ answer (2)
		[21]

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QUESTION 6

6.1	$A = P(1+i)^n$ $2x = x(1+0{,}054)^n$ $2 = (1{,}054)^n$ $\log(2) = n\log(1{,}054)$ $n = \frac{\log 2}{\log(1{,}054)}$ $n = 13{,}18$	✓ substitution into correct formula ✓ correct use of logs ✓ answer (3)
6.2	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $P = \frac{3300\left[1-\left(1+\frac{0{,}12}{12}\right)^{-60}\right]}{\frac{0{,}12}{12}}$ $= R148\,351{,}63$	✓ value of i ✓ value of n ✓ substitution into correct formula ✓ answer (4)
6.3	$i_{eff} = \left(1+\frac{0{,}05}{4}\right)^4 - 1$ $= 0{,}05094533691$ Rate = 5,09%	✓ value of n ✓ substitution into correct formula ✓ rate (3)
6.4	$10\,000\,000 = \frac{x\left[\left(1+\frac{0.15}{12}\right)^{384}-1\right]}{\frac{0.15}{12}}$ $125000 = x(116{,}9484518)$ $x = \text{R}1068{,}85$	✓ value of n ✓ substitution into correct formula ✓ answer (3)
		[13]

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QUESTION 7	**Penalty for notation error in 7.1 only.**	
7.1	$f(x) = -2x^2 + 1$ $f(x+h) = -2(x+h)^2 + 1$ $f(x+h) = -2x^2 - 4xh - 2h^2 + 1$ $f'(x) = \lim_{h\to 0} \frac{f(x+h)^2 - f(x)}{h}$ $f'(x) = \lim_{h\to 0} \frac{-2x^2 - 4xh - 2h^2 + 1 - (-2x^2 + 1)}{h}$ $f'(x) = \lim_{h\to 0} \frac{-2x^2 - 4xh - 2h^2 + 1 + 2x^2 - 1}{h}$ $f'(x) = \lim_{h\to 0} \frac{-4xh - 2h^2}{h}$ $f'(x) = \lim_{h\to 0} \frac{h(-4x - 2h)}{h}$ $= -4x$	✓ $-2x^2 - 4xh - 2h^2 + 1$ ✓ correct substitution into formula ✓ simplification ✓ common factor ✓ answer (5)
7.2.1	$f(x) = \frac{1}{2}x^2 - 5x^{-1}$ $f'(x) = x + 5x^{-2}$	✓ $-5x^{-1}$ ✓ x ✓ $5x^{-2}$ (3)
7.2.2	$D_x\left[\frac{-2x^2 + \sqrt[4]{x}}{x^2}\right]$ $= D_x\left[\frac{2x^2}{x^2} + \frac{x^{\frac{1}{4}}}{x^2}\right]$ $= D_x\left[-2 + x^{-\frac{7}{4}}\right]$ $= -\frac{7}{4}x^{-\frac{11}{4}}$	✓ $\sqrt[4]{x} = x^{\frac{1}{4}}$ ✓ $-2 + x^{-\frac{7}{4}}$ ✓ derivative (3)
		[11]

QUESTION 8

8.1

1	–1	5	8	–12
	0	–1	4	12
	–1	4	12	0

$(x-1)(-x^2+4x+12)=0$
$(x-1)(x^2-4x-12)=0$
$(x-1)(x-6)(x+2)=0$
$x=1 \text{ or } x=6 \text{ or } x=-2$
$\therefore C(6;0)$

✓ quadratic factor
✓ factors
✓ x-values
✓ answer (4)

8.2

$-3x^2+10x+8=0$
$3x^2-10x-8=0$
$(3x+2)(x-4)=0$
$x=-\frac{2}{3} \text{ or } x=4$
$f(4)=-(4)^3+5(4)^2+8(4)-12$
$=36$

✓ derivative
✓ $f'(x)=0$
✓ factors
✓ x-values
✓ answer (5)

8.3

$-6x+10>0$
$x<\frac{5}{3}$

✓ inequality
✓ answer (2)

8.4

$f'(x)=-3x^2+10x+8$
$f'(0)=8$
$-3x^2+10x+8=8$
$-3x^2+10x=0$
$x(-3x+10)=0$
$x=0 \text{ or } x=\frac{10}{3}$
$\therefore OG=\frac{10}{3}$

✓ $f'(0)=8$
✓ equating
✓ factors
✓ x-values
✓ value of OG (5)

8.5

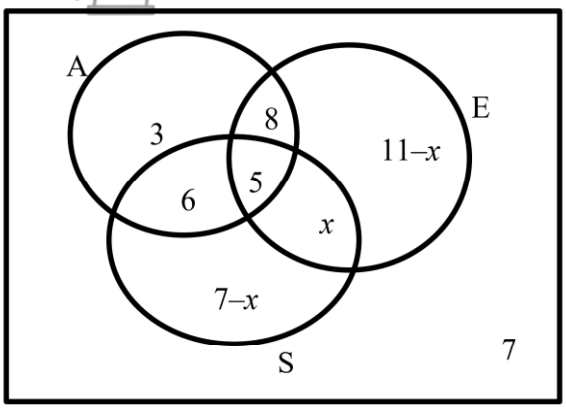
$f'(x)=-3x^2+10x+8$
$f'(5)=-3(5)^2+10(5)+8$
$=-17$
$m=\frac{1}{17}$

✓ substitution
✓ $f'(5)=-17$
✓ answer (3)

[19]

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QUESTION 9		
9.1	$m_{mn} = -\frac{b}{a}$ $y = -\frac{b}{a}x + b$	✓ m_{mn} ✓ answer (2)
9.2	Area of PTOR =TP x PR $= x\left(-\frac{b}{a}x + b\right)$ $= -\frac{b}{a}x^2 + bx$ $Max: -\frac{2b}{a}x + b = 0$ $x = b\left(\frac{a}{2b}\right)$ $x = \frac{a}{2}$ $y = -\frac{b}{a}\left(\frac{a}{2}\right) + b = \frac{b}{2}$ Midpoint of MN: $\left(\frac{0+a}{2}; \frac{0+b}{2}\right) = \left(\frac{a}{2}; \frac{b}{2}\right)$ $\therefore$ Max at midpoint	✓ substitution into area formula ✓ simplification ✓ derivative =0 ✓ value of x ✓ value of y ✓ midpoint (6)
		[8]

QUESTION 10		
10.1.1	$45 - 38 = 7$	✓ answer (1)
10.1.2	S = 45  	✓ 8 ✓ 6 ✓ $11-x$ ✓ $7-x$ (4)
10.1.3	$3+8+6+5+11-x+x+7-x+7=45$ $x=2$	✓ calculation ✓ answer (2)
10.1.4	$P(2languages)=\frac{6+8+2}{45}$ $=\frac{16}{45}$	✓ 16 ✓ $\frac{16}{45}$ (2)
10.2.1	$8! = 40320$	✓ 8! (1)
10.2.2	$3! \times 5! \times 6 = 4320$ **OR** $3! \times 6! = 4320$	✓3! ✓ 5! ✓ $3! \times 5! \times 6$ (3)
10.2.3	$\frac{5!}{8!}=\frac{1}{336}$	✓ 5! ✓ $\frac{5!}{8!}$ OR $\frac{1}{336}$ (2)
		[15]

TOTAL: 150

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