education

MPUMALANGA PROVINCE

REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PAPER 2

SEPTEMBER 2024

MARKING GUIDELINE / NASIENRIGLYNE

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MARKS/PUNTE: 150

This marking guideline consist of 16 pages.

Hierdie nasienriglyne bestaan uit 16 bladsye.

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NOTE:

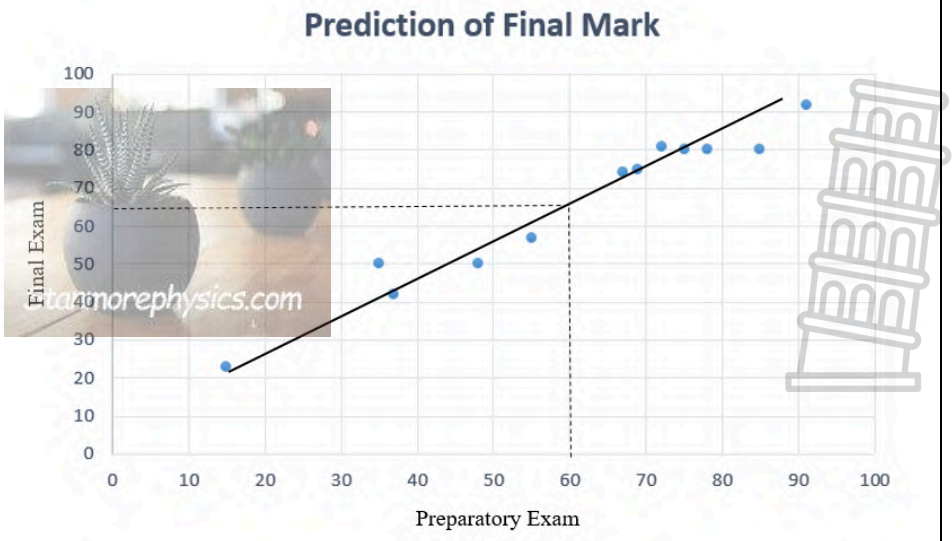
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guiselines.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyn van toepassing.*
- *Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.*

QUESTION 1/ VRAAG 1

Preparatory exam(x)	55	35	67	85	91	48	78	72	15	75	69	37
Final exam(y)	57	50	74	80	92	50	80	81	23	80	75	42

1.1	$a = 12{,}01$ $b = 0{,}88$ $y = 12{,}01 + 0{,}88x$	✓ value of a ✓ value of b ✓ Equation (3)
1.2		✓ any correct two points ✓ straight line joining the points ✓ passing through $(\bar{x}; \bar{y})$ (3)
1.3	$y = 12{,}01 + 0{,}88(46)$ $y = 52\%$ **OR** calculator $y = 52{,}50\%$	✓ Substitution ✓ Answer (2)
1.4	r = 0,98	✓ Answer (1)

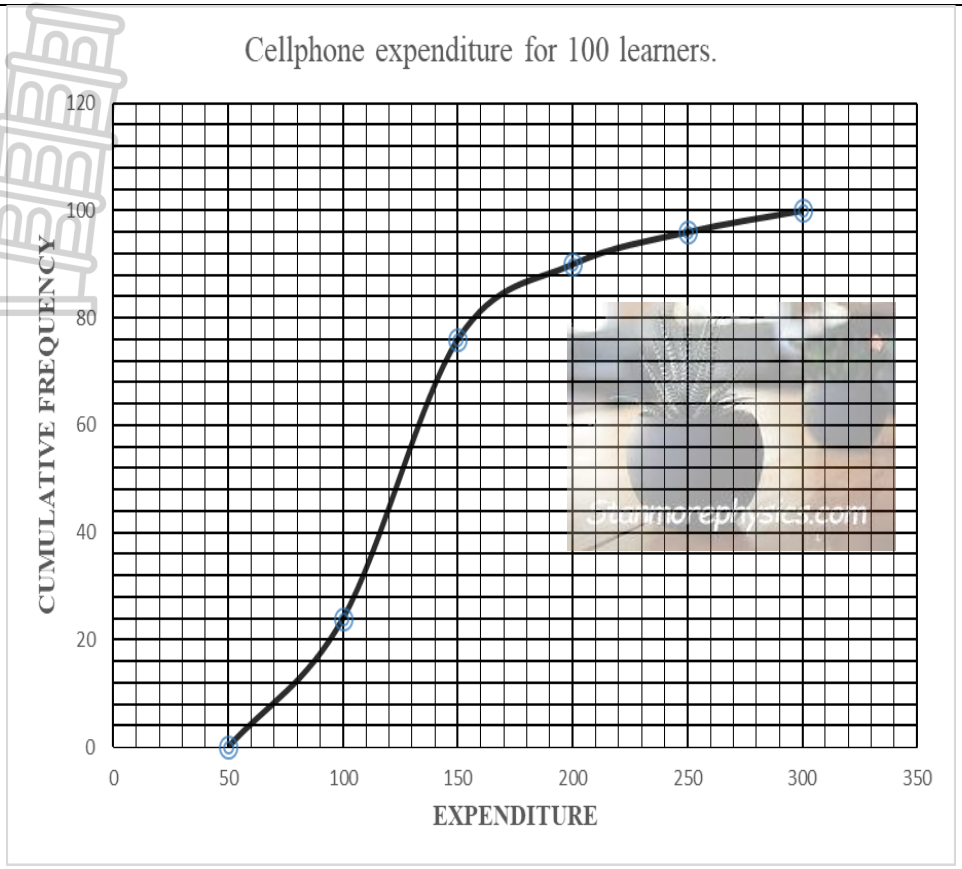
1.5	There's a very strong positive correlation between preparatory marks and final marks. Daar is 'n baie sterk positiewe korrelasie tussen die rekordeksamen en die finale punte.	✓ Answer (1)
1.6	No, the preparatory exam mark is the independent variable. Hence we cannot determine the preparatory mark using the final mark. Nee, die rekordeksamen is die onafhanklike veranderlike. Die finale punt kan dus nie gebruik word om die rekordeksamenpunt te voorspel nie.	✓ Answer ✓ Reason (2)
		[12]

QUESTION 2 / VRAAG 2

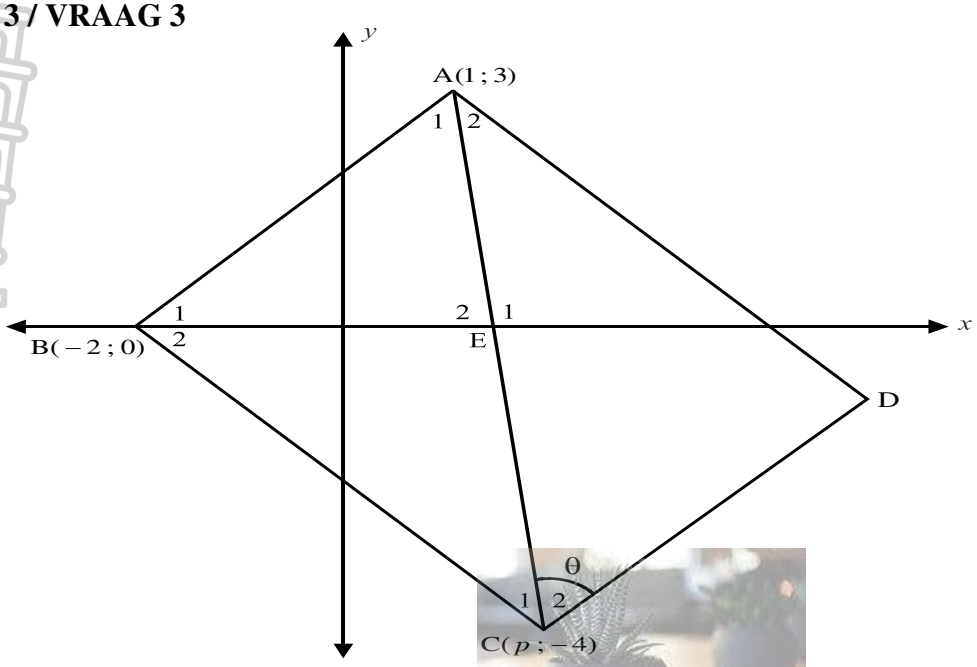
2.1

Expenditure(in rand) Uitgawes(in rand)	**Frequency Frekwensie**	**Cumulative frequency Kumulatiewe frekwensie**	
$50 \leq x < 100$	24	24	✓76 ; 90
$100 \leq x < 150$	52	76	
$150 \leq x < 200$	14	90	✓ 96 ; 100
$200 \leq x < 250$	**6**	96	
$250 \leq x < 300$	4	100	

(2)

2.2		✓ Correct points ✓ Grounding ✓ Shape (3)
2.3	$\bar{x} = \frac{(50\times 24)+(125\times 52)+(175\times 14)+(225\times 6)+(275\times 4)}{100}$ $= \frac{12600}{100}$ $= 126$	✓ Method ✓ $\div 100$ ✓ Answer (3)
		[8]

QUESTION 3 / VRAAG 3



3.1.1	$m_{AB} = \dfrac{3-0}{1-(-2)} = 1$	✓ substitution ✓ answer (2)
3.1.2	$AC^2 = (p-1)^2 + (-4-3)^2$ $\therefore (\sqrt{50})^2 = p^2 - 2p + 1 + 49$ $\therefore 50 = p^2 - 2p + 50$ $\therefore 0 = p^2 - 2p$ $\therefore 0 = p(p-2)$ $p = 0$ or $p = 2$ $\therefore p = 2$	✓ dist formula used ✓ correct substitution ✓ standard form ✓ working to $p = 2$ (4)
3.1.3	Midpoint of AB / Middelpunt van AB: $M = \left(\dfrac{-2+1}{2}; \dfrac{0+3}{2}\right) = \left(-\dfrac{1}{2}; \dfrac{3}{2}\right)$ Gradient of $\perp$ line through M: $m = -1$ Equation of perp bisector / Vergelyking van middelloodlyn: $y - \dfrac{3}{2} = -1\left(x + \dfrac{1}{2}\right)$ $\therefore y - \dfrac{3}{2} = -x - \dfrac{1}{2}$ $\therefore y = -x + 1$	✓ midpoint M ✓ grad of $\perp$ line ✓ sub into formula ✓ $y = -x + 1$ (4)

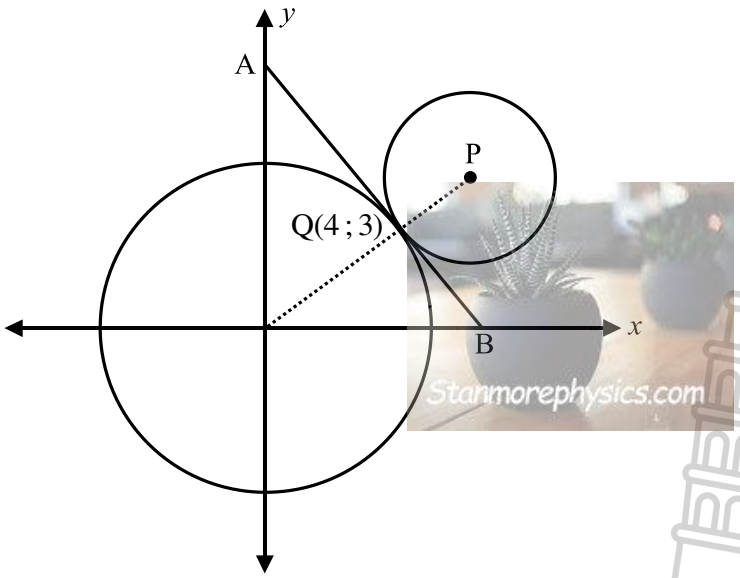
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3.1.4	D(5 ; −1)	✓ $x_D = 5$ ✓ $y_D = -1$ (2)
3.1.5	$\hat{B}_1 + \hat{B}_2 = 90°$ ABCD is a rectangle / reghoek $\therefore$ AC is a diameter / middellyn Centre is the midpoint of AC/Middelpunt van sirkel is middelpunt van AC. $\left(\frac{1+2}{2}; \frac{-4+3}{2}\right) = \left(\frac{3}{2}; -\frac{1}{2}\right)$ Radius is half of the length of AC/Radius is helfte van AC: $r = \frac{\sqrt{50}}{2}$ Equation of circle / Vergelyking van sirkel: $\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \left(\frac{\sqrt{50}}{2}\right)^2$ $\therefore \left(x - \frac{3}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{25}{2}$	✓ $\hat{B}_1 + \hat{B}_2 = 90°$ ✓ $\left(\frac{3}{2}; -\frac{1}{2}\right)$ ✓ $r = \frac{\sqrt{50}}{2}$ ✓ equation of circle (4)
3.1.6	$\hat{A}_1 = \theta$ alt ∠s = ; AB ∥ CD $\tan \hat{B}_1 = m_{AB}$ $\therefore \tan \hat{B}_1 = 1$ $\therefore \hat{B}_1 = 45°$ $\tan \hat{E}_1 = m_{AC} = \frac{-4-3}{2-1} = -7$ $\therefore \tan \hat{E}_1 = -7$ $\therefore \hat{E}_1 = 98{,}13010235°$ $\theta = 98{,}13010235° - 45°$ $\therefore \theta = 53°$	✓ $\hat{A}_1 = \theta$ ✓ $\hat{B}_1 = 45°$ ✓ $m_{AC} = -7$ ✓ $\hat{E}_1 = 98{,}13010235°$ ✓ $\theta = 53°$ (5)

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3.2	$3x+by=-2$ $\therefore by=-3x-2$ $\therefore y=-\frac{3}{b}x-\frac{2}{b}$ $-\frac{3}{b}x-\frac{2}{b}=-\frac{2}{3}x+2$ $\therefore -9x-6=2bx+6b$ Substitute $x=-3$: $\therefore -9(-3)-6=-2b(-3)+6b$ $\therefore 27-6=6b+6b$ $\therefore 21=12b$ $\therefore b=\frac{21}{12}=\frac{7}{4}$	✓ $y=-\frac{3}{b}x-\frac{2}{b}$ ✓ equating equations Stel vergelykings gelyk ✓ substituting $x=-3$ ✓ $b=\frac{7}{4}$ (4)
		[24]

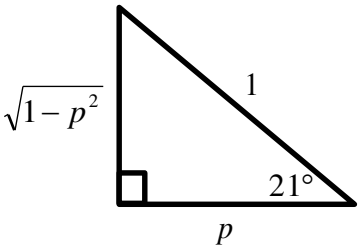
QUESTION 4 / VRAAG 4



4.1	$x^2-12x+y^2-9y+50=0$ $\therefore x^2-12x+36+y^2-9y+\frac{81}{4}=-50+36+\frac{81}{4}$ $\therefore (x-6)^2+\left(y-\frac{9}{2}\right)^2=\frac{25}{4}$	✓ $(x-6)^2$ ✓ $\left(y-\frac{9}{2}\right)^2$ ✓ $\frac{25}{4}$ (3)

4.2	$m_{OB} = \frac{3}{4}$ $\therefore m_{AB} = -\frac{4}{3}$ $y - 3 = -\frac{4}{3}(x - 4)$ $\therefore y - 3 = -\frac{4}{3}x + \frac{16}{3}$ $y = -\frac{4}{3}x + \frac{25}{3}$	✓ $m_{OB} = \frac{3}{4}$ ✓ $m_{AB} = -\frac{4}{3}$ ✓ substitution of point ✓ equation (4)
4.3	Circumference of small $= 2\pi\left(\frac{5}{2}\right) = 5\pi$ Circumference of large $= 2\pi(5) = 10\pi$ Two revolutions / Twee omwentelings .	✓ $r = \frac{5}{2}$ ✓ $r = 5$ ✓ 5π; 10π ✓ two revolutions (4)
4.4	$A\left(0\,;\frac{25}{3}\right)$ $B\left(\frac{25}{4}\,;0\right)$ $\therefore OA = \frac{25}{3}$ and $OB = \frac{25}{4}$ Area $\Delta AOB = \frac{1}{2} \times \frac{25}{4} \times \frac{25}{3} = \frac{625}{24}$ units^2	✓ $OA = \frac{25}{3}$ ✓ $OB = \frac{25}{4}$ ✓ calculating area (3)
4.5	Q(4 ; 3) C(–4 ; 3) $\therefore CQ = 8$ units	✓ C(–4 ; 3) ✓ $CQ = 8$ (2)
		[16]

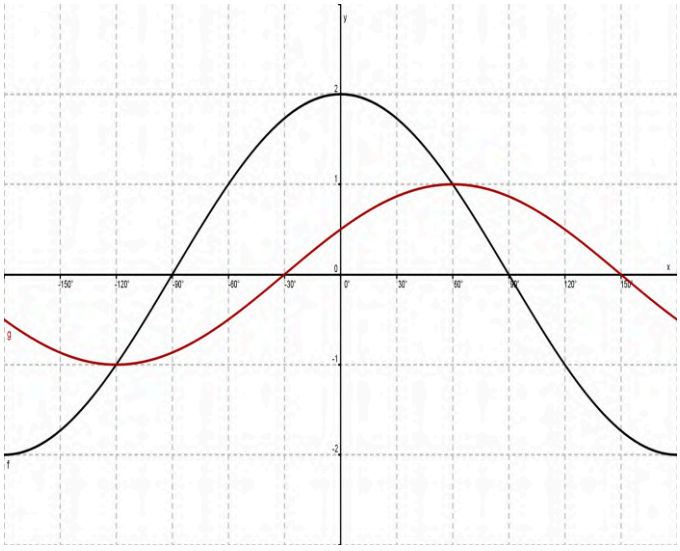
QUESTION 5 / VRAAG 5

5.1.1	$p^2 + y^2 = 1^2$ $y = \sqrt{1 - p^2}$ $\tan 201° = \tan(180° + 21°)$ $= \tan 21°$ 	✓ $y = \sqrt{1 - p^2}$ ✓ $\tan 21°$ ✓ $\frac{\sqrt{1-p^2}}{p}$ (3)

	$= \frac{\sqrt{1-p^2}}{p}$	
5.1.2	$\sin 42° = \sin 2(21°)$ $= 2\sin 21° \cos 21°$ $= 2\left(\sqrt{1-p^2}\right)(p)$ $= 2p\sqrt{1-p^2}$	✓ $\sin 2(21°)$ ✓ Identity ✓ Answer. (3)
5.1.3	$\cos(30^\circ + 21^\circ) = \cos 30^\circ \cos 21^\circ - \sin 30^\circ \sin 21^\circ$ $= \frac{\sqrt{3}}{2}(p) - \frac{1}{2}\left(\sqrt{1-p^2}\right)$ $= \frac{p\sqrt{3} - \sqrt{1-p^2}}{2}$	✓ $\cos(30^\circ + 21^\circ)$ ✓ Compound angle ✓ Substitution ✓ Answer (3)
5.2	$\frac{\sin 210° \cos 510°}{\cos 315° \sin(-135°)}$ $= \frac{(-\sin 30°)(\cos 150°)}{(\cos 45°)(-\sin 135°)}$ $= \frac{(-\sin 30°)(-\cos 30°)}{(\cos 45°)(-\sin 45°)}$ $= \frac{\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)}$ $= -\frac{\sqrt{3}}{2}$	✓ $-\sin 30°$ ✓ $-\cos 30°$ ✓ $\cos 45°$ ✓ $-\sin 45°$ ✓ $-\frac{1}{2}$ ✓ $-\frac{\sqrt{2}}{2}$ ✓ $-\frac{\sqrt{3}}{2}$ (7)
5.3	$\frac{\cos\theta - \cos 2\theta + 2}{3\sin\theta - \sin 2\theta}$ $= \frac{\cos\theta - (2\cos^2\theta - 1) + 2}{3\sin\theta - 2\sin\theta\cos\theta}$ $= \frac{\cos\theta - 2\cos^2\theta + 1 + 2}{\sin\theta(3 - 2\cos\theta)}$ $= \frac{3 + \cos\theta - 2\cos^2\theta}{\sin\theta(3 - 2\cos\theta)}$ $= \frac{(3 - 2\cos\theta)(1 + \cos\theta)}{\sin\theta(3 - 2\cos\theta)}$ $= \frac{1 + \cos\theta}{\sin\theta}$ $= RHS$	✓ $2\cos^2\theta - 1$ ✓ $2\sin\theta\cos\theta$ ✓ $3 + \cos\theta - 2\cos^2\theta$ ✓ $(3 - 2\cos\theta)(1 + \cos\theta)$ ✓ $\sin\theta(3 - 2\cos\theta)$ (5)

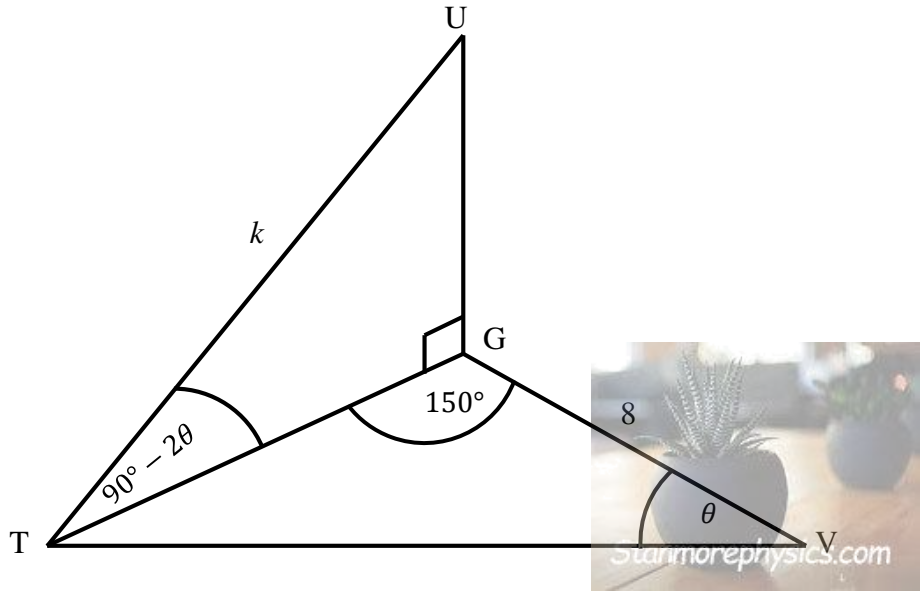
5.4	$\sin\theta\sin\frac{3\theta}{2}+\cos\frac{3\theta}{2}\cos\theta=-\frac{\sqrt{3}}{2}$ $\cos\frac{3\theta}{2}\cos\theta+\sin\theta\sin\frac{3\theta}{2}=-\frac{\sqrt{3}}{2}$ $\cos\left(\frac{3\theta}{2}-\theta\right)=-\frac{\sqrt{3}}{2}$ $\cos\left(\frac{\theta}{2}\right)=-\frac{\sqrt{3}}{2}$ $\frac{\theta}{2}=150°+k.360°\ k\in\mathbb{Z}\ OR\ \frac{\theta}{2}=-150°+k.360k\in\mathbb{Z}$ $\theta=300°+k.720°k\in\mathbb{Z}\qquad\theta=-300°+k.720°k\in\mathbb{Z}$ **OR** $ref\angle=30°$ $\frac{\theta}{2}=180°-30°+k.360°\boldsymbol{or}\frac{\theta}{2}=180°+30°+k.360°k\in\mathbb{Z}$ $\theta=300°+k.720°\ k\in\mathbb{Z}\ \boldsymbol{or}\ \theta=420°+k.720°\ \ k\in\mathbb{Z}$	✓ $\cos\left(\frac{3\theta}{2}-\theta\right)$ ✓ $\frac{\theta}{2}=\pm150°+k.360°$ ✓ $\theta=300°+k.720°$ ✓ $-300°+k.720°$ (4) **OR** ✓ $\cos\left(\frac{3\theta}{2}-\theta\right)$ ✓ $\frac{\theta}{2}=180°\pm+k.360°$ ✓ $300°+k.720°$ ✓ $420°+k.720°$ (4)
5.5.1	Maximum value is 1 and minimum value is -1	✓ Answer (1)
5.5.2	Range of both graphs is/waardevers van beide grafieke: $-1\le y\le 1$ $\sin\theta.\cos\beta=-1$ $\sin\theta=1\ and\ \cos\beta=-1\ \ or\ \ \sin\theta=-1\ \ and\ \ \cos\beta=-1$ $\theta=90°\ \ and\ \ \beta=-180°\ \ or\ \ \theta=270°\ \ and\ \beta=0°$	✓ $\theta=90°$ ✓ $\beta=-180°$ ✓ $\theta=270°$ ✓ $\beta=0°$ (4)
		[30]

QUESTION 6 / VRAAG 6

6.1		$f(x)$ ✓ $x-intercepts$ ✓ $y-intercept$ ✓ Shape. $g(x)$ ✓ $x-intercepts$ ✓ $y-intercept$ ✓ Shape. (6)

6.2.1	$360°$	✓ Answer (1)
6.2.2	$0°$	✓ Answer (1)
6.3.3	$90° < x < 150°$ OR $x \in (90°; 150°)$	✓ Notation ✓ Endpoints. (2)
		[10]

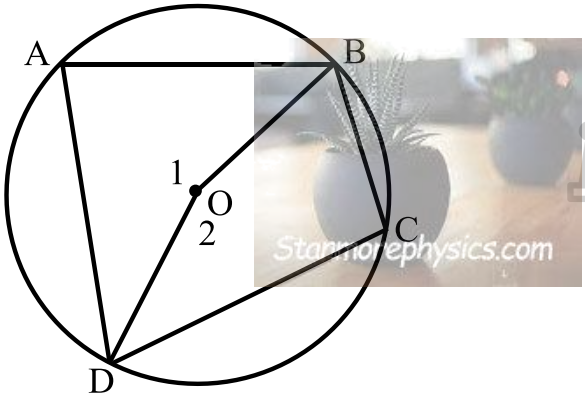
QUESTION 7 / VRAAG 7

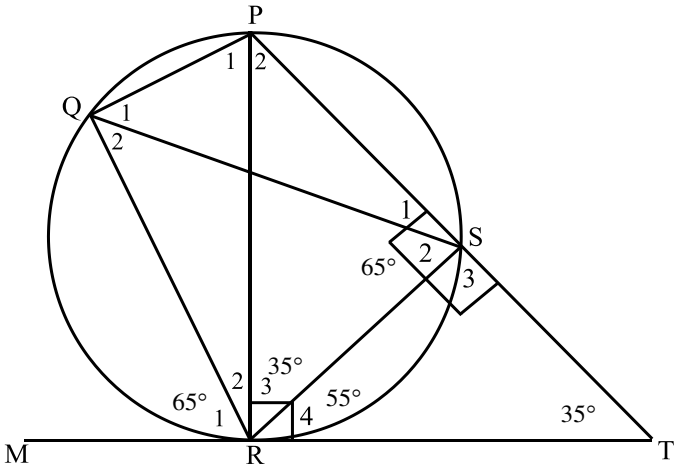


7.1	$\hat{TUG} = 2\theta$	✓Answer. (1)
7.2	$\frac{TG}{\sin 2\theta} = \frac{k}{\sin 90^\circ}$ $k \sin 2\theta = TG \times 1$ $\therefore TG = k \sin 2\theta$ OR $\cos(90^\circ - 2\theta) = \frac{TG}{k}$ $\sin 2\theta = \frac{TG}{k}$ $\therefore TG = k \sin 2\theta$	✓ Sine rule. ✓ $\sin 90^\circ = 1$ **OR** ✓ Trig ratio. ✓ $\sin 2\theta$ (2)

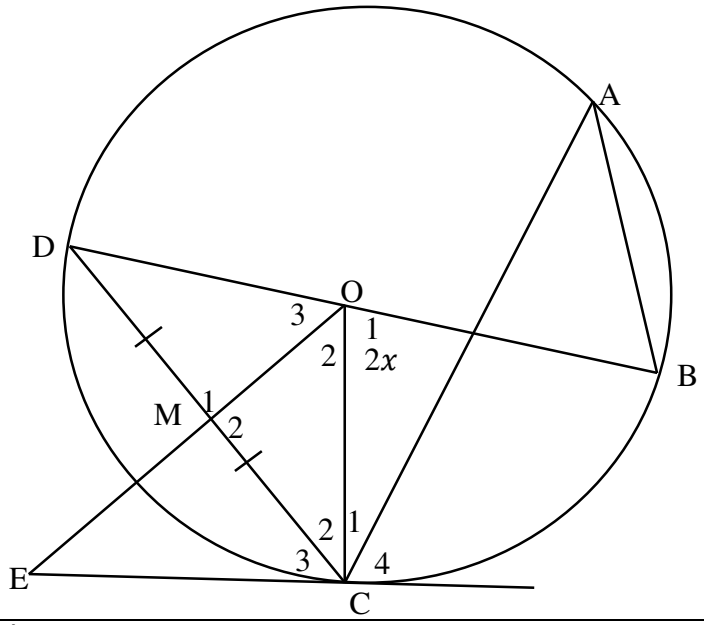
7.3	$\frac{TV}{\sin 150^\circ} = \frac{TG}{\sin\theta}$ $\frac{TV}{\frac{1}{2}} = \frac{TG}{\sin\theta}$ $TV = \frac{\frac{1}{2}TG}{\sin\theta}$ $TV = \frac{\frac{1}{2}(k\sin 2\theta)}{\sin\theta}$ $TV = \frac{\frac{1}{2}(2k\sin\theta\cos\theta)}{\sin\theta}$ $\therefore TV = k\cos\theta$	✓ $\frac{TV}{\sin 150^\circ} = \frac{TG}{\sin\theta}$ ✓ $\frac{1}{2}$ ✓ $TV = \frac{\frac{1}{2}TG}{\sin\theta}$ ✓ $2\sin\theta\cos\theta$ (4)
7.4	$\triangle TGV = \frac{1}{2}TG.GV\sin 150°$ $= \frac{1}{2}k\sin 2\theta.8.\frac{1}{2}$ $= 2k\sin 2\theta$ **OR** $\triangle TGV = \frac{1}{2}TV.GV.\sin\theta$ $= \frac{1}{2}k\cos\theta.8.\sin\theta$ $= 4\cos\theta.\sin\theta$ $= 2k(2\cos\theta\sin\theta)$ $= 2k\sin 2\theta$	✓ Substitution. ✓ Method. (2) **OR** ✓ Substitution. ✓ Method. (2)
		[09]

QUESTION 8 / VRAAG 8

8.1	 $\hat{O}_1 = 2\hat{C}$ $\angle$ at centre $= 2\times\angle$at circ $\hat{O}_2 = 2\hat{A}$ $\angle$ at centre $= 2\times\angle$at circ	✓ construction ✓ $\hat{O}_1 = 2\hat{C}$ ✓ $\hat{O}_2 = 2\hat{A}$

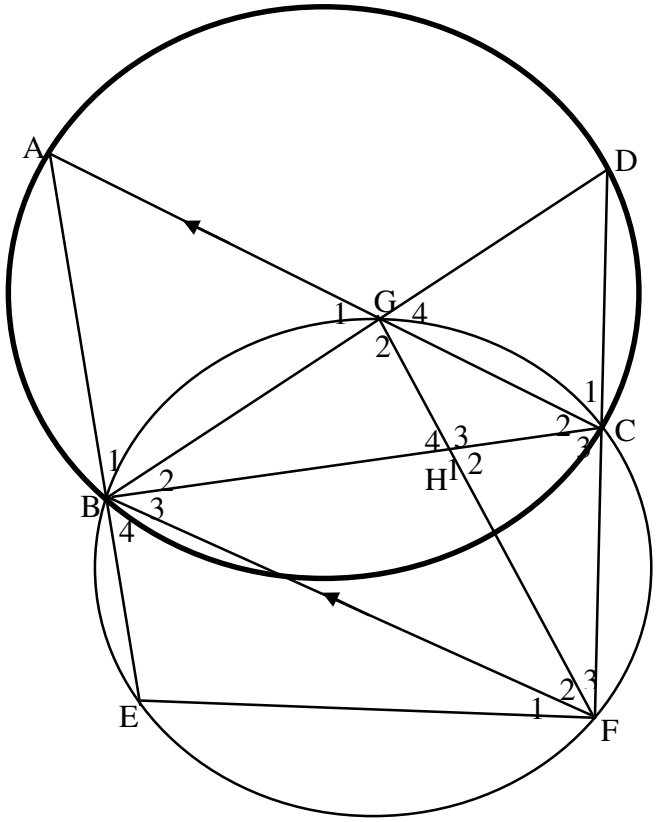
	$\hat{O}_1 + \hat{O}_2 = 2\hat{C} + 2\hat{A}$ $360° = 2\hat{C} + 2\hat{A}$ ∠s round a point $\therefore 180° = \hat{C} + \hat{A}$ $\therefore \hat{A} + \hat{C} = 180°$	✓ Writing the statement $360° = 2\hat{C} + 2\hat{A}$ and hence $\hat{A} + \hat{C} = 180°$ ✓ two correct reasons (5)
8.2		
8.2.1	$\hat{R}_1 = 65°$ tan chord theorem	✓ S /R (1)
8.2.2	$\hat{R}_3 + \hat{R}_4 = 90°$ tan ⊥ radius $\therefore \hat{R}_4 = 55°$	✓ S ✓ R (2)
8.2.3	$\hat{S}_1 + \hat{S}_2 = 90°$ ∠ in semi-circle $\hat{T} = 35°$ ext ∠ of Δ	✓ S ✓ R ✓ S (3)
		[11]

QUESTION 9 / VRAAG 9



9.1	$\hat{O}_1 = \hat{A}$ [∠ at the centre is = 2x ∠ at the circumference] $2x = \hat{A}$ $\hat{A} = x$ But $\hat{A} = \hat{D} = x$ [$\angle's$ in the same segment] $\hat{D} = \hat{C}_2 = x$ [$\angle's$ opp = sides]	✓ S ✓ R ✓ S ✓R ✓S ✓ R (6)
9.2	$\hat{M}_2 = 90°$ [∠ from centre to midpoint of a chord] $\therefore\ \hat{O}_2 = 90° - x$ [sum of $\angle's$ in a △]	✓ S ✓ R ✓ R (3)
9.3	$\hat{O}_2 = 90° - x$ [proved in 9.2] $\therefore\ \hat{C}_3 = 90° - x$ [tan ⊥ rad] $\therefore\ \hat{O}_2 = \hat{C}_3$ [both = $90° - x$] ∴ EC is a tangent to circle passing through M, C & O. Converse tan-chord theorem.	✓S ✓ R ✓ S ✓ R (4)
9.4	$\hat{C}_3 = 90° - x$ [proven in 9.3] $\hat{O}_3 = 180° - (90° - x) - 2x$ [$\angle's$ on a straight line] $\therefore \hat{O}_3 = 90° - x$ $\therefore \hat{O}_3 = \hat{C}_3$ [both = $90° - x$] ∴ DOCE is a cyclic quadrilateral. Converse $\angle's$ in the same segment.	✓S ✓ R ✓ R (3)
		[16]

QUESTION 10/ VRAAG10



10.1.1	Tan-chord theorem	✓ Answer (1)
10.1.2	Corresp. $\angle s$; BF ∥ AC	✓ Answer (1)
10.2.1	$\hat{C}_2 = \hat{B}_3$ [alt.∠s; AC ∥ BF] $\hat{B}_3 = \hat{G}_3$ [∠s in the same segment] $\hat{G}_3 = \hat{F}_2$ [Alt. ∠s; AC ∥ BF] $\therefore \hat{B}_3 = \hat{F}_2$ [both = $\hat{G}_3$] $\therefore$ BH = FH [sides opp. = ∠s]	✓ S/R ✓ S/R ✓ S ✓ R (4)
10.2.2	In ΔBEF and ΔDGF $\hat{E} = \hat{G}_4$ [ext. ∠ of cyclic quad.] $\hat{B}_4 = \hat{A}$ [alt. ∠s; AC ∥ BF] $\hat{A} = \hat{D}$ [∠s in the same segment] $\therefore \hat{B}_4 = \hat{A}$ $\hat{F}_1 = \hat{F}_3$ [sum of ∠s in a Δ] ΔBEF ⫼ ΔDGF [∠∠∠]	✓ S/R ✓ S/R ✓ R (3)

10.2.3	$\hat{B}_2 = \hat{F}_3$ $\hat{H}_2 = \hat{H}_4$ $\therefore \hat{G}_2 = \hat{C}_3$ $\Delta BGH \|\|\| \Delta FCH$ $\frac{BG}{FC} = \frac{BH}{FH}$ $\therefore FH.BG = BH.FC$	[$\angle$s in the same segment] [vert. opp $\angle s$] [sum of $\angle s$ in a Δ] [$\angle\angle\angle$]	✓ S/R ✓ S/R ✓ R ✓ S (4)
			[13]

TOTAL/ TOTAAL:		**150**