education

MPUMALANGA PROVINCE

REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12



MATHEMATICS P1

JUNE 2024

MARKS: 150

TIME: 3 HOURS

This question paper consists of 10 pages including 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. The question paper consists of 8 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.

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QUESTION 1

1.1 Given : $f(x) = (x+4)(3-x)$ and $g(x) = x^2 - 49$

Solve for x if:

1.1.1 $g(x) = 0$ (2)

1.1.2 $f(x) > 0$ (3)

1.1.3 $f(x) = 5$, rounded off to THREE decimal places. (4)

1.2 Solve x:

1.2.1 $\sqrt{x+2} = x-4$ (4)

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1.2.2 $2x^{-\frac{5}{3}} = 64$ (3)

1.3 Solve for x and y simultaneously:

$-2y + x = -1$ and $x^2 - 7 - y^2 = -y$ (6)

1.4 Determine the values of p, for which the equation $2^x = 1 - 2p$ will have real solutions. (2)

1.5 Given $M = \sqrt{\dfrac{9-3p}{p+1}}$

Determine the value(s) of p for which M will be:

1.5.1 Undefined. (1)

1.5.2 Rational (give only ONE integer). (1)

[26]

QUESTION 2

2.1 Consider the pattern: $5; -2; -7; -10; .....$

2.1.1 Determine an expression for the n^{th} term of the sequence. (4)

2.1.2 Show that the sequence will never have a term with a value less than -11. (4)

2.2 Consider the sequence $\frac{1}{181}; \frac{2}{181}; \frac{3}{181}; \frac{4}{181}; .....; \frac{180}{181}$

Calculate the sum of all the terms of the above sequence. (3)

2.3 Given the geometric sequence: $1{,}21; \ 1{,}331; \ 1{,}4641; \ .....$

2.3.1 Determine the 12^{th} term of the sequence (do not round off). (3)

2.3.2 Calculate the sum of the first 12 terms of the sequence (round off to 3 decimal places). (3)

2.4 Write the following series in sigma notation:

$$\frac{1}{3}+\frac{4}{9}+\frac{7}{27}+\frac{10}{81}+.....+\frac{22}{6561}$$ (4)

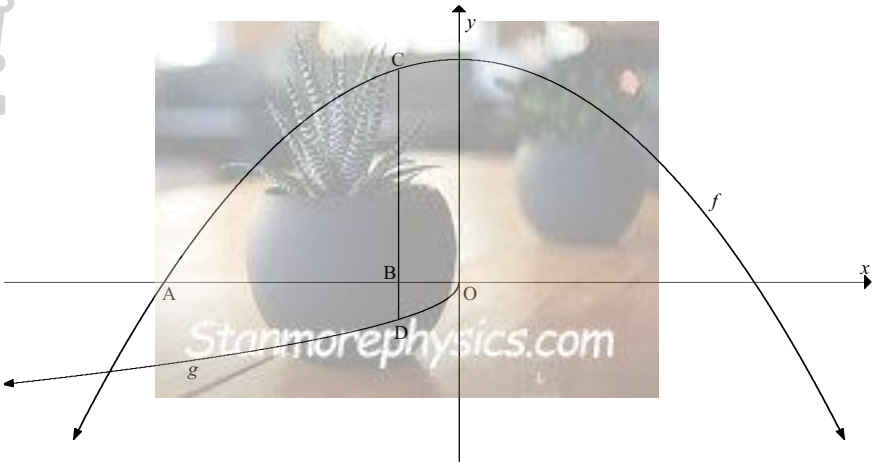
2.5 Consider the geometric series: $(2x-4)+(4x^2-16)+.....$

For which value(s) of x will the series converge? (4)

[25]

QUESTION 3

Given: $f(x) = -4x^2 + 6$ and $g(x) = -2\sqrt{-x};\ x \leq 0.$ The graphs of *f* and g are sketched. A is an x-intercept of f and B is a point between O and A. The straight line CBD, with C on f and D on *g*, is parallel to the *y*-axis.



3.1 Determine the *x*-coordinate of A, correct to TWO decimal places. (2)

3.2 Show that $f(x) \neq 8$. (2)

3.3 Write down the length of CBD in terms of x, where x is the x-coordinate of B. (2)

3.4 Determine the maximum length of CD. (6)

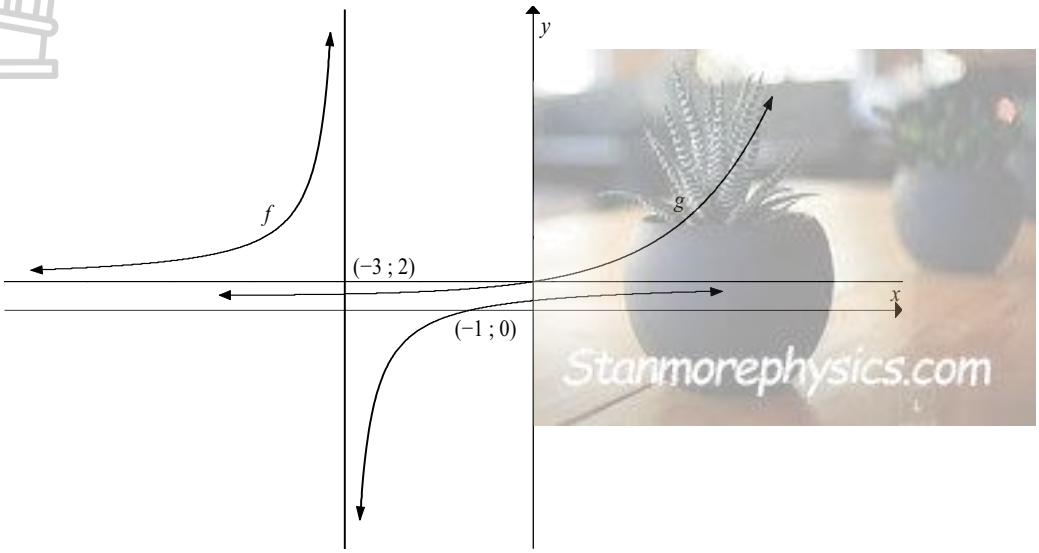
3.5 Write the equation of h, the inverse of g, in the form $y = ...$ (4)

3.6 Write the equation of k, the reflection of g about the x-axis, in the form $y = ...$ (1)

3.7 If $f(x) + k = 0$ has two distinct real roots, determine the value(s) of k. (2)
[19]

QUESTION 4

The sketch below represents the graphs of $f(x)=\frac{a}{x+p}+q$ and $g(x)=2^x+1$. The point $A(-3;2)$ is where the asymptotes of f intersect. The graph of f intersects the x-axis at $(-1;0)$.



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4.1 Write down the equations of the asymptotes of f. (2)

4.2 Write down the coordinates of the y-intercept of g. (2)

4.3 Determine the equation of f. (4)

4.4 Write the equation of the axis of symmetry of f in the form $y=mx+c$ if $m<0$. (2)

4.5 Write down the domain of $4f(x-2)$. (2)

4.6 Write the equation of h, if h is the graph of g that is translated one unit down. (1)

4.7 Determine the equation of h^{-1}, the inverse of h. (2)

4.8 Write down the range of h^{-1}. (1)

4.9 For which values of x if $f(x).g'(x)\leq 0$. (2)

[18]

QUESTION 5

5.1 How long will it take for an article to depreciate to zero if the depreciation rate is 12,5% per annum? (2)

5.2 Bandile invests R25 000 into a savings account, earning 7,3% compounded interest per annum.

5.2.1 Calculate the nominal interest rate, compounded quarterly, rounded off to 3 decimal digits. (4)

5.2.2 Using the effective rate, how much money will be in his account after 66 months? (3)

5.3 Zanele wants to give each of her two kids R100 000 in the year when each is 18 years old. They will be 18 in respectively 3 years and 5 years from now. To be able to do so, she immediately invests an amount in her savings account at 6% p.a. compounded monthly. How much money must she invest now? Round off your answer to the nearest hundred. (4)

5.4 Nathan deposits R50 000 into a savings account, earning 8,4% p.a. compounded quarterly. After two years the interest rate changes to 7,9% p.a. compounded half-yearly. How much money will be in his account 7 years after he started the savings plan? (4)

[17]

QUESTION 6

6.1 Given: $f(x) = 2 - 3x^2$

Determine $f'(x)$ from FIRST PRINCIPLES. (5)

6.2 Determine:

6.2.1 $f'(x) = \frac{x^4}{2} + x^0 - p\,t^2$ (2)

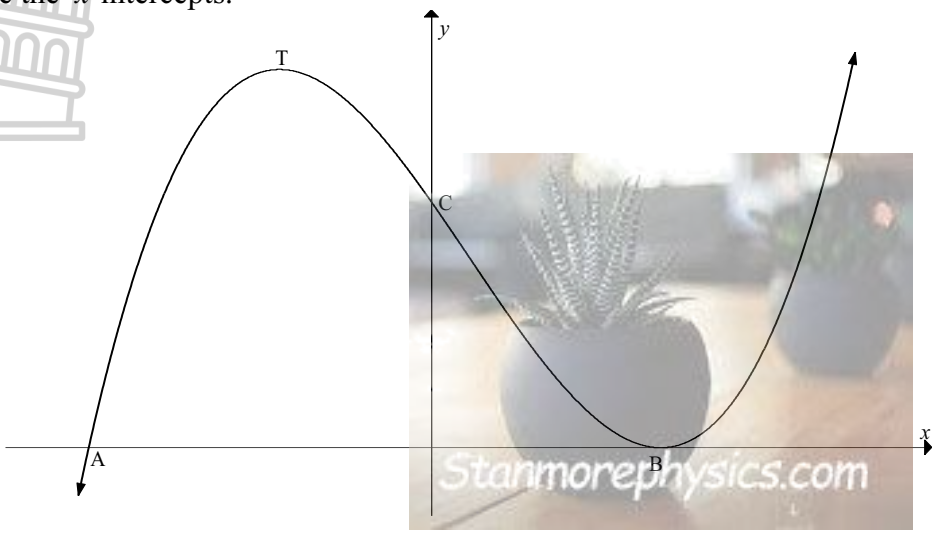
6.2.2 $D_x\left[\frac{-3x^3 + \sqrt[5]{x^2}}{x^2}\right]$ (4)

6.2.3 $\frac{dy}{dx}$ if $y = \frac{8x^3 - 125}{2x - 5}$ (3)

[14]

QUESTION 7

Given $f(x) = x^3 - x^2 - px + q.$ Point T $\left(-\frac{4}{3}; 18\frac{14}{27}\right)$ is a stationary point and $f(-3) = 0$. A and B are the x-intercepts.



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7.1 Show by calculation that $p = 8$ and $q = 12$. **(4)**

7.2 Calculate the length of AB. (4)

7.3 For which values of x is the function concave up? (3)

7.4 Determine the equation of the tangent to the curve at the point where $f(x)$ changes concavity. (4)

7.5 If $f(x) = k$ has one distinct real root, determine the value(s) of k. (2)

7.6 Give the value(s) of x for which $f(x) > 0$. (2)

[19]

QUESTION 8

8.1 The sample space S is the set of natural numbers less than 10. A is the set of even numbers and B is the set of prime numbers.

8.1.1 Draw a Venn-diagram to represent the information. (4)

8.1.2 Are events A and B exhaustive? Give a reason for your answer. (2)

8.1.3 What is $n(A \cup B)'$? (1)

8.1.4 What is the probability of drawing an even number **and** a prime number? (1)

8.2 The South African Traffic Service is doing a clamp down on speeding. During a recent speeding trap they collected the following data:

	Speeding	**Not speeding**	**TOTAL**
Male	398	217	615
Female	205	180	385
TOTAL	603	397	1000

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Are events being a male and speeding independent? Motivate your answer with the necessary calculations. (4)

[12]

TOTAL MARKS: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni)$$

$$A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In $\triangle ABC$: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \triangle ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha\ \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha\ \sin\beta$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$= 1 - 2\sin^2\alpha$$

$$= 2\cos^2\alpha - 1$$

$$\sin 2\alpha = 2\sin\alpha\ \cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_1 - x)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A\ of\ B) = P(A) + P(B) - P(A\ en\ B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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