education

MPUMALANGA PROVINCE

REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

JUNE 2023

MARKS: 150

TIME: 3 hours

Stanmorephysics

This question paper consists of 8 pages and a one-page information sheet.

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
   Answer ALL the questions.
2. Show clearly ALL the calculations, diagrams, graphs, etcetera, which you have used in determining the answers.
3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. Answers only will not necessarily be awarded full marks.
6. Number the answers correctly according to the numbering system used in this question paper.
7. Diagrams and graphs are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

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QUESTION 1

1.1 Solve for x.

1.1.1 $(2+x)(-x+4)=0$ (2)

1.1.2 $3x^2 = 2x+4$ (Correct to 2 decimal places) (4)

1.1.3 $x-2\sqrt{x-1}=4$ (6)

1.1.4 $-x-12>-x^2$ (4)

1.2 Solve the following equations simultaneously:

$x^2-xy-5y^2=-5$ and $x+2y=1$ (6)

1.3 Determine the values of t for which the equation

$5^x = 2-t$ will have real solutions. (3)

[25]

QUESTION 2

Given the arithmetic series $3 + 8 + 13 + \ldots\ldots$(to 253 terms)

2.1 Write down the 4th term of the series. (1)

2.2 Calculate the 253rd term of the series. (3)

2.3 Express the series in sigma notation. (2)

2.4 Calculate the sum of the series. (2)

2.5 How many terms of the series are divisible by 4? (4)

[12]

QUESTION 3

3.1 Given the quadratic sequence: 1; 3; 7; p; …

3.1.1 Calculate the value of p. (3)

3.1.2 Determine the n^{th} term of the sequence. (4)

3.1.3 The first difference between two consecutive terms is 62. Calculate the values of these terms. (3)

3.2 Determine the largest integer value of m if $\sum_{k=-2}^{m}(2.2^{k+2}) < 2046$ (5)

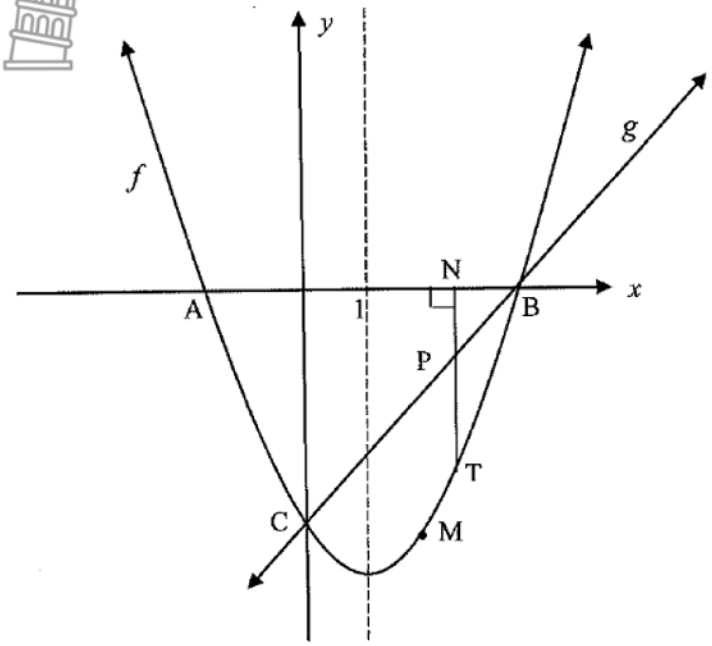
[15]

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QUESTION 4

The following sketch shows the graphs of $f(x) = x^2 + bx + c$ and $g(x) = ax + q$.

The graph of f intersects the x-axis at A(–2 ; 0) and at B. C is the y-intercept of both f and g. The axis of symmetry of f is $x = 1$.



4.1 Determine

4.1.1 The coordinates of B (2)

4.1.2 The equation of f in the form $y = x^2 + bx + c$ (2)

4.1.3 The values of a and q. (4)

4.1.4 The coordinates of the turning point of f. (2)

4.2 M is the reflection of C in the axis of symmetry of f. Write down the coordinates of M. (2)

4.3 For which values of x is $f(x) > g(x)$? (2)

4.4 Write down the coordinates of the turning point of $y = -f(x - 2)$ (2)

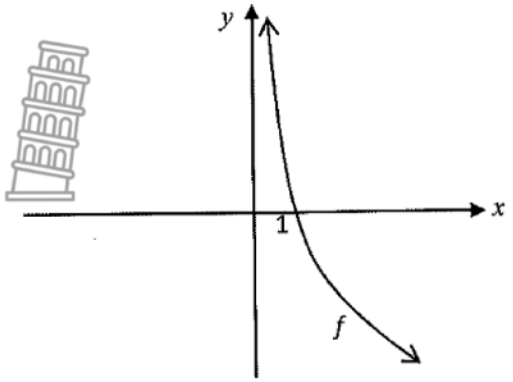
4.5 N is a point on the x-axis and T is a point on f such that TN is perpendicular to the x-axis. TN intersects g at P. Calculate the maximum length of TP. (4)

[20]

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QUESTION 5

Sketched below is the graph of $f(x) = -\log_3 x$



5.1 Write down the domain of f. (1)

5.2 Write down the equation of f^{-1} in the form $y=\ldots$. (2)

5.3 Describe the transformation from f^{-1} to h if $h(x)=3^{-x}-5$ (2)

5.4 Use the graph of f to solve for x if $k(x)$ is the reflection of f about the x-axis and $k(x) \geq 3$. (4)

[9]

QUESTION 6

The function f defined by $f(x) = \frac{a}{x+p} + q$ has the following properties.

- The range of f is $y \in R, y \neq 2$
- The axis of symmetry with a positive gradient is $y = x + 1$
- The graph of f passes through $(0;-4)$

6.1 Write down the value of q. (1)

6.2 Calculate the values of a and p. (4)

6.3 Sketch a neat graph of this function. Your graph must include the intercepts with the axes and asymptotes if any. (5)

[10]

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QUESTION 7

7.1 Thelma purchased a new car. The bank offered her a loan at an effective interest rate of 10,4% p.a.
Determine the nominal rate compounded monthly that she is required to pay. (4)

7.2 An investor bought shares in a certain company, but found that his money had become half of its original value after a period of 4 years. What was the annual rate of decay, on a reducing balance method, for the shares he had bought? (4)

7.3 Thini borrowed R80 000, exactly five years ago, at 21% interest p.a. She paid back R25 000 exactly three years ago and R55 000 exactly a year ago. She wants to settle her remaining debt today.

7.3.1 Calculate the value of the loan to date. (2)

7.3.2 How much must Thini pay to settle the debt today? (5)

[15]

QUESTION 8

8.1 Given $f(x) = 4x - 2x^2$, determine $f'(x)$ using FIRST PRINCIPLES. (5)

8.2 Determine:

8.2.1 $D_x\left[x^4 - \frac{1}{x^2}\right]$ (3)

8.2.2 $\frac{dy}{dx}$ if $x\sqrt{y} = x^2 - 4x$ (4)

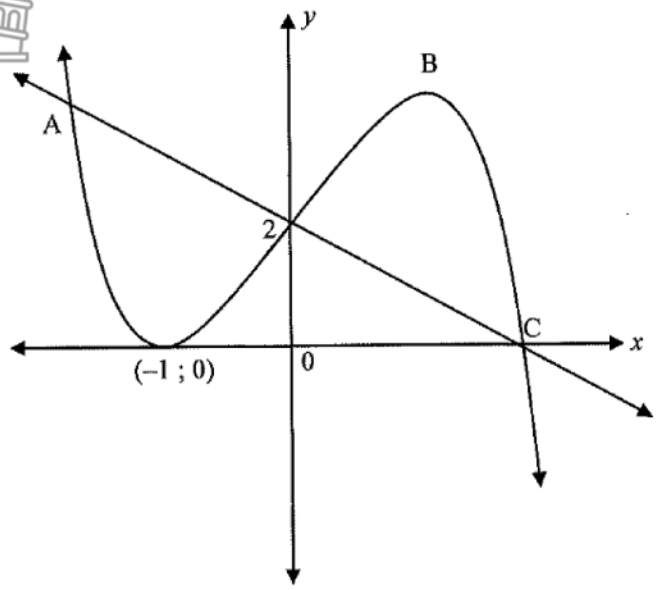
8.3 Find the equation of the tangent to the graph of $f(x) = x^3 - 6x^2$ at which the gradient of the tangent to f is equal to 36 and $x < 0$. (5)

[17]

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QUESTION 9

The graph below represents the functions f and g with $f(x) = ax^3 + cx + 2$ and $g(x) = -x + 2$. C and $(-1; 0)$ are the x-intercepts of f. The graph of f and g intersect at A and C.



9.1 Determine the coordinates of C. (1)

9.2 Show by calculation that $a = -1$ and $c = 3$. (4)

9.3 Determine the coordinates of B, the turning point of f. (3)

9.4 Find the x-coordinate of the point of inflection of f. (2)

9.5 Write down the values of k for which $f(x) = k$ will have only one root. (2)

9.6 Write down the values of x for which $f'(x) < 0$. (2)

[14]

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QUESTION 10

10.1 Given that A and B are independent events, P(B only) = 0,4

P(A and B) = 0,1 ; P(A only) = x and P(not A or B) = y.

Calculate:

10.1.1 The values of x and y. (4)

10.1.2 The probability that at least one of A or B occurs. (3)

10.2 In a city 10% of all crimes are violent. 95% of all violent crimes are reported, buy only 45% of non- violent crimes are reported.

10.2.1 Draw a tree diagram showing all the possible outcomes. (3)

10.2.2 What is the probability that a random crime will be reported? (3)

[13]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + in) \qquad A = P(1 - in) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1};\ r \neq 1 \qquad S_\infty = \frac{a}{1 - r};\ -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i} \qquad P = \frac{x[1 - (1 + i)^{-n}]}{i} \qquad f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{Area of } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta \qquad \cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$$

$$\cos 2A = \begin{cases} \cos^2 A - \sin^2 A \\ 1 - 2\sin^2 A \\ 2\cos^2 A - 1 \end{cases} \qquad \sin 2A = 2\sin A.\cos A$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})}$$

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