KWAZULU-NATAL PROVINCE

EDUCATION

REPUBLIC OF SOUTH AFRICA

MATHEMATICS P1

COMMON TEST

SEPTEMBER 2024

MARKING GUIDELINES

NATIONAL SENIOR CERTIFICATE

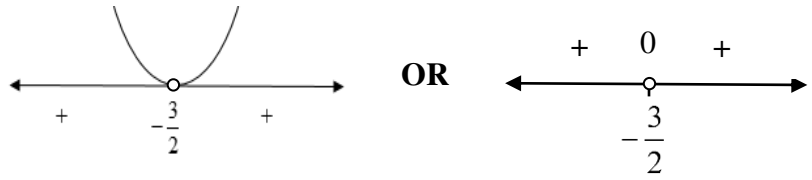
GRADE 12

MARKS: 150

These marking guidelines consist of 17 pages.

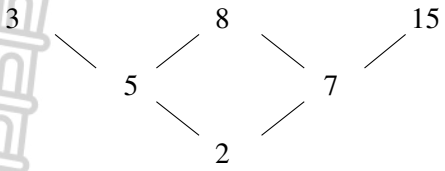
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QUESTION 1

1.1.1	$x(x-5)=0$ $x=0$ or $x=5$ Answer only: Full marks	✓A factors ✓A answer (0) ✓ CA answer (5) (3)
1.1.2	$x=\frac{-2\pm\sqrt{(2)^2-4(5)(-6)}}{2(5)}$ $x=0{,}91$ or $-1{,}31$ Answer only: 2/3 **Penalise for incorrect rounding only in this question.**	✓A substitution into formula ✓CA answer ✓CA answer (3)
1.1.3	$2^x(2-3.2^{-1}+1)=12$ $2^x\left(3-\frac{3}{2}\right)=12$ $2^x\left(\frac{3}{2}\right)=12$ $2^x=8$ $2^x=2^3$ **OR** $x=\log_2 8$ $\therefore x=3$	✓A factors ✓CA prime bases (or use of logarithms) ✓CA answer (3)
1.1.4	$(2x+3)(2x+3)>0$ CV: $-\frac{3}{2}$  $x\in\mathrm{R}$, but $x\neq-\frac{3}{2}$ **OR** $x<-\frac{3}{2}$ or $x>-\frac{3}{2}$ **OR** $\left(-\infty;-\frac{3}{2}\right)$ or $\left(-\frac{3}{2};\infty\right)$	✓A factors ✓A $x\in\mathrm{R}$; ✓CA $x\neq-\frac{3}{2}$ (3)

1.2	$2x-y+1=0$ $y=2x+1$ $x^2+x(2x+1)-(2x+1)=3x-2$ $x^2+2x^2+x-2x-1-3x+2=0$ $3x^2-4x+1=0$ $(3x-1)(x-1)=0$ $x=\frac{1}{3}$ or 1 $y=2\left(\frac{1}{3}\right)+1=\frac{5}{3}$ $y=2(1)+1=3$	✓A $y=2x+1$ ✓CA substitution ✓CA standard form ✓CA x-values ✓CA y-values (5)
	OR $2x-y+1=0$ $2x=y-1$ $x=\frac{y-1}{2}$ $\left(\frac{y-1}{2}\right)^2+y\left(\frac{y-1}{2}\right)-y=3\left(\frac{y-1}{2}\right)-2$ $\frac{y^2-2y+1}{2}+\frac{y^2-y}{2}-y=\frac{3y-3}{2}-2$ $y^2-2y+1+2y^2-2y-4y=6y-6-8$ $3y^2-8y+1=6y-14$ $3y^2-14y+15=0$ $(3y-5)(y-3)=0$ $y=\frac{5}{3}$ or 3 $x=\frac{\frac{5}{3}-1}{2}=\frac{1}{3}$ $x=\frac{3-1}{2}=1$	**OR** ✓A $x=\frac{y-1}{2}$ ✓CA substitution ✓CA standard form ✓CA y-values ✓CA x-values (5)
1.3	$x^2-2x-3=y^2-2y-3$ $x^2-y^2-2x+2y=0$ $(x+y)(x-y)-2(x-y)=0$ $(x-y)(x+y-2)=0$ $x-y=0$ $\quad x+y-2=0$ N/A $\quad x=2-y$	✓A expanding ✓CA standard form ✓A factors $(x+y)(x-y)$ ✓CA factors ✓CA answer $x=2-y$ only (5)
		[22]

QUESTION 2

2.1.1	 $2a = 2$ $a = 1$ $5 = 3(1) + b$ $b = 2$ $3 = 1 + 2 + c$ $c = 0$ $T_n = n^2 + 2n$	✓ A $a = 1$ ✓CA $b = 2$ ✓CA $c = 0$ ✓CA answer (4)
2.1.2	$n^2 + 2n = 1700$ $n^2 + 2n - 1700 = 0$ $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $n = \frac{-2 \pm \sqrt{2^2 - 4(1)(1700)}}{2(1)}$ $n = 40, 24$ or $n = -42, 24$ n is not a natural number and therefore 1700 is not a term in the sequence.	✓CA equating T_n to 1700 ✓CA substitution into formula ✓CA values of n ✓CA 1700 is not a term in the sequence (4)

2.2	$(1-2)(1+2)+(3-4)(3+4)+(5-6)(5+6)+(7-8)(7+8)+.....$ $.....+(399-400)(399+400)$ $=(-1)(3)+(-1)(7)+(-1)(11)+(-1)(15)+.....(-1)(799)$ $=-3-7-11-15.....-799$	✓A factors ✓A arithmetic series
	$a=-3 \quad d=4$ $T_n=a+(n-1)d$ $-799=-3+(n-1)4$ $n=200$	✓CA $n=200$
	$S_n=\frac{n}{2}[2a+(n-1)d]$ OR $S_n=\frac{n}{2}(a+l)$ $S_{200}=\frac{200}{2}[2(-3)+(200-1)(-4)]$ $S_{200}=\frac{200}{2}(-3-799)$ $=-80200$ $=-80200$	✓CA substitution in S_n formula ✓CA answer (5)
	OR	**OR**
	$1^2+3^2+5^2+7^2+.....+399^2$ $=(2n-1)^2$	
	$-2^2-4^2-6^2-8^2-.....-400^2$ $=-(2n)^2$ $=(2n-1)^2-(2n)^2$ $=4n^2-4n+1-4n^2$ $=-4n+1$	✓A $(2n-1)^2-(2n)^2$ ✓A $-4n+1$
	$2n=400$ **OR** $2n-1=399$ $n=200$	✓CA $n=200$
	$\sum_{n=1}^{200}(-4n-1)=-3-7-11.....$ $S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{200}=\frac{200}{2}[2(-3)+(200-1)(-4)]$ $=-80200$	✓CA substitution in S_n formula ✓CA answer (5)
		[13]

QUESTION 3

3.1.1	$81;\ m;\ \frac{m}{3};\ .....$ $\frac{m}{81}=\frac{\frac{m}{3}}{m}$ $\frac{m}{81}=\frac{1}{3}$ OR $m^2=\frac{81m}{3}$ $3m=81$ $m^2=27m$ $m=27$ $m=27$ Answer only: Full marks	✓A $\frac{m}{81}=\frac{\frac{m}{3}}{m}$ ✓A answer (2)
3.1.2	$\sum_{t=1}^{9}81\left(\frac{1}{3}\right)^{t-1}=81+27+9+3+....$ $a=81$ $r=\frac{1}{3}$ $S_n=\frac{a\left(r^n-1\right)}{r-1}$ $S_9=\frac{81\left(\left(\frac{1}{3}\right)^9-1\right)}{\frac{1}{3}-1}$ $=\frac{9841}{81}=121,49$	✓A value of a ✓A value of r ✓CA substitute into formula ✓CA answer (4)
3.2.1	$a=\frac{12}{5};\quad d=\frac{3}{5};\quad T_n=\frac{333}{5}$ $T_n=a+(n-1)d$ $\frac{333}{5}=\frac{12}{5}+(n-1)\left(\frac{3}{5}\right)$ $333=12+3n-3$ $333n=324$ $n=108$	✓A $d=\frac{3}{5}$ ✓A substitute $\frac{333}{5}$ into formula ✓ CA substitute d into formula ✓CA answer (4)

3.2.2	$\frac{12}{5}; 3; \frac{18}{5}; \frac{21}{5}; \frac{24}{5}; \frac{27}{5}; 6; \ldots\ldots\ldots 66; \frac{333}{5}$ Terms that are integers: 3 ; 6 ; 9 ; ……; 66 $T_n = a + (n-1)d$ $66 = 3 + (n-1)3$ $66 = 3 + 3n - 3$ $3n = 66$ $n = 22$ **OR** The following terms are integers: T_2 ; T_7; .....T_{107} Sequence: 2 ; 7 ; 12 ; ……. 107 $T_n = a + (n-1)d$ $107 = 2 + (n-1)5$ $107 = 2 + 5n - 5$ $5n = 110$ $n = 22$	✓A identifying one more term that is an integer ✓A correct sequence ✓CA substitution ✓CA answer **OR** ✓A identifying the position of one more term that is an integer ✓A correct sequence ✓CA substitution ✓CA answer (4)
		[14]

QUESTION 4

4.1	$x^2-2x-3=0$ $(x+1)(x-3)=0$ $x=-1$ or $x=3$ $\therefore \mathrm{P}(-1\,;0)\quad \mathrm{Q}(3\,;0)$	✓A factors ✓CA answer ✓CA answer (3)
4.2	At turning point: $x=\frac{-b}{2a}$ $=\frac{-(-2)}{2(1)}$ $=1$ $y=1^2-2(1)-3=-4$ $\therefore \mathrm{T}(1\,;-4)$ **OR** $h(x)=(x^2-2x+1)-3-1$ $h(x)=(x-1)^2-4$ $\therefore \mathrm{T}(1\,;-4)$ Answer only: 2/3 **OR** x-value at TP: $=\frac{x_1+x_2}{2}$ $=\frac{-1+3}{2}$ $=1$ $y=1^2-2(1)-3=-4$ $\therefore \mathrm{T}(1\,;-4)$ **OR** $h'(x)=2x-2=0$ $x=1$ $y=1^2-2(1)-3=-4$ $\therefore \mathrm{T}(1\,;-4)$	✓A substitution ✓CA x-value ✓CA y-value (3) **OR** ✓A completing the square ✓CA x-value ✓CA y-value (3) **OR** ✓CA average value of x-intercepts ✓CA x-value ✓CA y-value (3) **OR** ✓A derivative = 0 ✓CA x-value ✓CA y-value (3)
4.3	$x=0$	✓A answer (1)
4.4	$g(x)=\frac{a}{x}-2$ Substitute $\mathrm{P}(-1\,;0)$: $0=\frac{a}{-1}-2$ $a=-2$	✓A $q=-2$ ✓CA substitution ✓CA answer (3)

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4.5	Yes, it is a function For every x-value, there are only one y-value OR It passes the vertical line test.	✓A answer ✓A justification (2)
4.6	$x^2-2x-3=\frac{-2}{x}-2$ $x^3-2x^2-3x=-2-2x$ $x^3-2x^2-x+2=0$ $x^2(x-2)-(x-2)=0$ $(x-2)(x^2-1)=0$ $(x-2)(x-1)(x+1)=0$ $x=-1$ or $x=1$ or $x=2$ At K: $x=2$ $y=\frac{-2}{2}-2=-3$ K(2 ; –3)	✓CA equating ✓CA simplification ✓CA factors ✓CA x value ✓CA y value (5)
4.7	$x\in(-1;0)$ or $x\in(1;2)$ **OR** $-1<x<0$ or $1<x<2$	✓CA✓CA $x\in(-1;0)$ ✓CA $x\in(1;2)$ (3) **OR** ✓CA✓CA $-1<x<0$ ✓CA $1<x<2$ (3)
		[20]

QUESTION 5

5.1	$-2x+3=0$ $x=\frac{3}{2}$ Answer only: Full marks	✓A equating to 0 ✓A $x=\frac{3}{2}$ (2)
5.2	$-2^{-x+1}.x+6.2^{-x-1}<0$ $-2.2^{-x}x+6.2^{-x}.2^{-1}<0$ $-2x.2^{-x}+3.2^{-x}<0$ $2^{-x}(-2x+3)<0$ $\frac{3}{2}<x\leq 3$	✓A splitting exponents ✓A factorisation ✓CA ✓CA answer (4)
5.3	$p:\ x=2^{-y}$ $y=-\log_2 x$ **OR** $p:\ x=\left(\frac{1}{2}\right)^y$ $y=\log_{\frac{1}{2}} x$ Answer only: Full marks	✓A swapping x and y ✓CA answer (2) **OR** ✓A swapping x and y ✓CA answer (2)

5.4	range of f^{-1} = domain of f $-4 \le y \le 3$	✓✓ A A answer (2)
5.5	Intersection between $y = -2x + 3$ and $y = x$: $-2x + 3 = x$ $-3x = -3$ $x = 1$ $y = 1$ **OR** $f^{-1}: x = -2y + 3$ $y = \frac{-x+3}{2}$ $-2x + 3 = \frac{-x+3}{2}$ $x = 1$ $y = 1$ **OR** $x = \frac{-x+3}{2}$ $x = 1$ $y = 1$	✓A $-2x + 3 = x$ ✓A x-value ✓A y-value (3) **OR** ✓A $-2x + 3 = \frac{-x+3}{2}$ ✓A x-value ✓A y-value (3) **OR** ✓A $x = \frac{-x+3}{2}$ ✓A x-value ✓A y-value (3)
		[12]

QUESTION 6

6.1	$A = P(1+i)^n$ $13460 = 6500\left(1+\frac{i}{4}\right)^{16}$ $\left(1+\frac{i}{4}\right)^{16} = \frac{13460}{6500}$ $1+\frac{i}{4} = \sqrt[16]{\frac{13460}{6500}}$ $i = 0,1862 = 18,6\%$ $r = 18,6$	✓A $n = 16$ ✓CA substitution ✓CA $1+\frac{i}{4} = \sqrt[16]{\frac{13460}{6500}}$ ✓CA answer (4)
6.2	$F = \frac{x\left[(1+i)^n - 1\right]}{i}; \quad n = 17$ $65000 = \frac{x\left[\left(1+\frac{0,08}{12}\right)^{17} - 1\right]}{\frac{0,08}{12}}$ $x = \frac{65000 \times \frac{0,08}{12}}{\left(1+\frac{0,08}{12}\right)^{17} - 1}$ $x = \text{R}3623,67$	✓A $n = 17$ ✓CA substitution ✓CA answer (3)
6.3.1	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $\text{R}650\,000 = \frac{7000\left[1-\left(1+\frac{0,11}{12}\right)^{-n}\right]}{\frac{0,11}{12}}$ $\frac{650000 \times \frac{0,11}{12}}{7000} = 1-\left(1+\frac{0,11}{12}\right)^{-n}$ $\left(1+\frac{0,11}{12}\right)^{-n} = \frac{25}{168}$ $\log_{\left(1+\frac{0,11}{12}\right)}\left(\frac{25}{168}\right) = -n$ $n = 208,7788941$ 208 instalments of R7 000	✓A substitution ✓CA use of logarithms ✓CA value of n ✓CA answer (4)

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6.3.2	Outstanding balance at $T_{208} = \dfrac{7000\left[1-\left(1+\dfrac{0,11}{12}\right)^{-0,7788941}\right]}{\dfrac{0,11}{12}}$ $= \text{R5 } 408,18$ Final payment $= 5408,18\left(1+\dfrac{0,11}{12}\right)$ $= \text{R5 } 457,75$ **OR** Outstanding balance at T_{208} $= 650000\left(1+\dfrac{0,11}{12}\right)^{208} - \dfrac{7000\left[\left(1+\dfrac{0,11}{12}\right)^{208}-1\right]}{\dfrac{0,11}{12}}$ $= \text{R5 } 408,18$ Final payment $= 5408,18\left(1+\dfrac{0,11}{12}\right)$ $= \text{R5 } 457,75$	✓CA value of n ✓CA substitution in present value formula ✓CA compounding ✓CA answer (4) **OR** ✓CA $= 650000\left(1+\dfrac{0,11}{12}\right)^{208}$ ✓CA $- \dfrac{7000\left[\left(1+\dfrac{0,11}{12}\right)^{208}-1\right]}{\dfrac{0,11}{12}}$ ✓CA compounding ✓CA answer (4)
		[15]

QUESTION 7

Penalise once only for incorrect notation in 7.1.

7.1	$f(x)=-x^2+x$ $f(x+h)=-\left(x+h\right)^2+(x+h)=-x^2-2xh-h^2+x+h$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-x^2-2xh-h^2+x+h-\left(-x^2+x\right)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-x^2-2xh-h^2+x+h+x^2-x}{h}$ $=\lim\limits_{h\to 0}\dfrac{-2xh-h^2+h}{h}$ $=\lim\limits_{h\to 0}\dfrac{h\left(-2x-h+1\right)}{h}$ $=\lim\limits_{h\to 0}\left(-2x-h+1\right)$ $=-2x+1$	✓A value of $f(x+h)$ ✓CA substitution into correct formula ✓CA simplifying ✓CA factors ✓CA answer (5)
7.2.1	$y=x^3\left(4-x^{-3}\right)$ $y=4x^3-1$ $\dfrac{dy}{dx}=12x^2$	✓A product ✓CA answer (2)
7.2.2	$f(x)=\dfrac{2x^2+3}{\sqrt{x}}$ $f(x)=\dfrac{2x^2}{x^{\frac{1}{2}}}+\dfrac{3}{x^{\frac{1}{2}}}$ $f(x)=2x^{\frac{3}{2}}-3x^{-\frac{1}{2}}$ $f'(x)=3x^{\frac{1}{2}}-\dfrac{3}{2}x^{-\frac{3}{2}}$	✓A $2x^{\frac{3}{2}}$; ✓A $-3x^{-\frac{1}{2}}$ ✓CA $3x^{\frac{1}{2}}$; ✓CA $-\dfrac{3}{2}x^{-\frac{3}{2}}$ (4)
		[11]

QUESTION 8

8.1	$0=-\frac{1}{2}x+2$ $\frac{1}{2}x=2$ $x=4$ $\therefore \text{Q}(4\ ;\ 0)$	✓A equating to zero ✓A answer (2)
8.2	$n=-4$	✓CA answer (1)
8.3	$0=(x-1)^2(x-4)$ $x=1$ or 4 PQ $=4-1=3$ units	✓CA values of x. ✓CA answer (2)
8.4	$f(x)=x^3-6x^2+9x-4$ $f'(x)=3x^2-12x+9$ $3x^2-12x+9=0$ $x^2-4x+3=0$ $(x-1)(x-3)=0$ $x=3$ $y=(3)^2-6(3)^2+9(3)-4=-4$ $\therefore \text{S}(3\ ;-4)$	✓CA multiplying out ✓CA derivative ✓CA equating to zero ✓CA value of x ✓CA value of y (5)
8.5	f is concave down at $x=0$.	✓A concave down (1)
8.6	To determine where $f'(x)=$ gradient of h: $\therefore 3x^2-12x+9=-\frac{1}{2}$ $3x^2-12x+\frac{19}{2}=0$ $x=\frac{-(-12)\pm\sqrt{(-12)^2-4(3)\left(\frac{19}{2}\right)}}{2(3)}$ $x=1,09 \quad \text{or} \quad x=2,91$	✓CA equating derivative to $-\frac{1}{2}$ ✓CA standard form ✓CA substitution ✓CA ✓CA values of x (5)
		[16]

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QUESTION 9

9.1	$h'(t) = 20 - 10t$ $0 = 20 - 10t$ $t = 2$ seconds Max height: $h(2) = -5(2)^2 + 20(2) + 1$ $= 21$metres	✓A derivative ✓CA equating derivative to zero ✓CA value of t ✓CA substitution ✓CA answer (5)
9.2	$1 + 20t - 5t^2 = 0$ $5t^2 - 20t - 1 = 0$ $t = \frac{20 \pm \sqrt{(-20)^2 - 4(5)(-1)}}{2(5)}$ $t = -0,05$ or $4,05$ $t = 4,05$ seconds	✓A equating h to zero ✓A values of t ✓CA answer (3)
9.3	$h'(t) = 20 - 5t$ $h'(1,5) = 20 - 10(1,5)$ $= 5$ m/s	✓CA substitution in $h'(t)$ ✓CA answer (2)
		[10]

QUESTION 10

10.1.1	P(Female) $= \frac{648}{864} = \frac{3}{4}$	✓A answer (1)
10.1.2	P(Female) × P(Positive) $= \frac{648}{864} \times \frac{108}{864}$ $= \frac{3}{32} = 0,09$ P(Female and Positive) $= \frac{81}{864}$ $= \frac{3}{32} = 0,09$ P(Female and Positive) = P(Female) × P(Positive) Events are independent. Testing positive is independent of gender. **OR** P(Male) × P(Positive) $= \frac{216}{864} \times \frac{108}{864}$ $= \frac{1}{32} = 0,03$ P(Male and Positive) $= \frac{27}{864}$ $= \frac{1}{32} = 0,03$ P(Male and Positive) = P(Male) × P(Positive) Events are independent. Testing positive is independent of gender.	✓A P(Female) × P(Positive) ✓A $0,09375$ or $0,09$ ✓A P(Female and Positive) = $0,09375$ or $0,09$ ✓A conclusion (4) **OR** ✓A P(Male) × P(Positive) ✓A $0,03125$ or $0,03$ ✓A P(Male and Positive) = $0,03125$ or $0,03$ ✓A conclusion (4)
10.2.1	$20 \times 20 \times 10 \times 10 \times 20 \times 20$ $= 16\,000\,000$ number plates	✓A $20 \times 20 \times 10 \times 10 \times 20 \times 20$ ✓A answer (2)
10.2.2	$20 \times 19 \times 10 \times 9 \times 18 \times 17$ $= 10\,465\,200$ number plates Also accept: $18 \times 17 \times 10 \times 9 \times 16 \times 15$ $= 6\,609\,600$	✓A $20 \times 19 \times 10 \times 9 \times 18 \times 17$ ✓A answer (2)
10.2.3	$\frac{2 \times 1 \times 10 \times 9 \times 18 \times 17}{10\,465\,200}$ $= \frac{1}{190}$ Also accept: $\frac{2 \times 1 \times 10 \times 9 \times 16 \times 15}{6\,609\,600}$ $= \frac{1}{153}$	✓A numerator ✓A denominator ✓CA answer (3)

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10.3	Let the number of red sweets be x. P(at least one green sweet) = 1 − P(no green sweets) $= 1-\left(\frac{x}{10}\right)^4$ $1-\left(\frac{x}{10}\right)^4 = 0,9744$ $\left(\frac{x}{10}\right)^4 = 1-0,9744$ $\left(\frac{x}{10}\right)^4 = 0,0256$ $\left(\frac{x}{10}\right) = \frac{2}{5}$ $x = 4$ There are 6 green sweets in the bag.	✓A $1-\left(\frac{x}{10}\right)^4$ ✓A equating to 97,44% ✓CA value of x ✓CA answer (4)
		[16]

TOTAL: 150

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