KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

PREPARATORY EXAMINATION

SEPTEMBER 2022

MARKING GUIDELINE

Stanmorephysics.com

MARKS: 150

TIME: 3 hours

These marking guideline consists of 15 pages.

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NOTE:

- If a candidate answered a QUESTION TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answer in order to solve a problem is unacceptable.

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason.)**
R	**A mark for a correct reason** **(A reason mark may only be awarded if the statement is correct.)**
S/R	**Award a mark if the statement AND reason are both correct.**

QUESTION 1

40	47	48	51	53	57	58	58	59	59
60	60	60	60	61	62	63	64	66	69

1.1.1	$\bar{x} = \frac{1155}{20}$ $\bar{x} = 57,75\ kg$ Answer only: full marks	A✓ 1155 CA✓ answer **Penalty 1 mark for rounding here for the entire paper**	(2)
1.1.2	$\sigma = 6,73702 \approx 6,74\ kg$	A✓ answer	(1)
1.2	$(\bar{x} - \sigma\ ;\ \bar{x} + \sigma\ )$ $(57,75 - 6,74\ ;\ 57,75 + 6,74)$ limit $= (51,01\ ;\ 64,49)$ $\therefore$ 14 boys	CA✓ interval CA✓ answer	(2)
1.3.1	22	A✓ answer	(1)
1.3.2	$\bar{x} = \frac{1320}{22}$ $\bar{x} = 60\ kg$	CA✓ based on 1.3.1	(1)
1.4	$\bar{x} = \frac{5x + 1155}{25} = 60$ $5x + 1155 = 1500$ $5x = 345$ $x = 69\ kg$	CA✓ $\frac{5x+1155}{25}$ CA✓ equation CA✓ simplification CA✓ answer	(4)
			[11]

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QUESTION 2

2.1

DISTANCE, d (in km)	FREQUENCY	CUMULATIVE FREQUENCY
$0 < d \leq 5$	8	**8**
$5 < d \leq 10$	41	**49**
$10 < d \leq 15$	63	**112**
$15 < d \leq 20$	52	**164**
$20 < d \leq 25$	41	**205**
$25 < d \leq 30$	38	**243**
$30 < d \leq 35$	7	**250**
TOTAL	**250**	

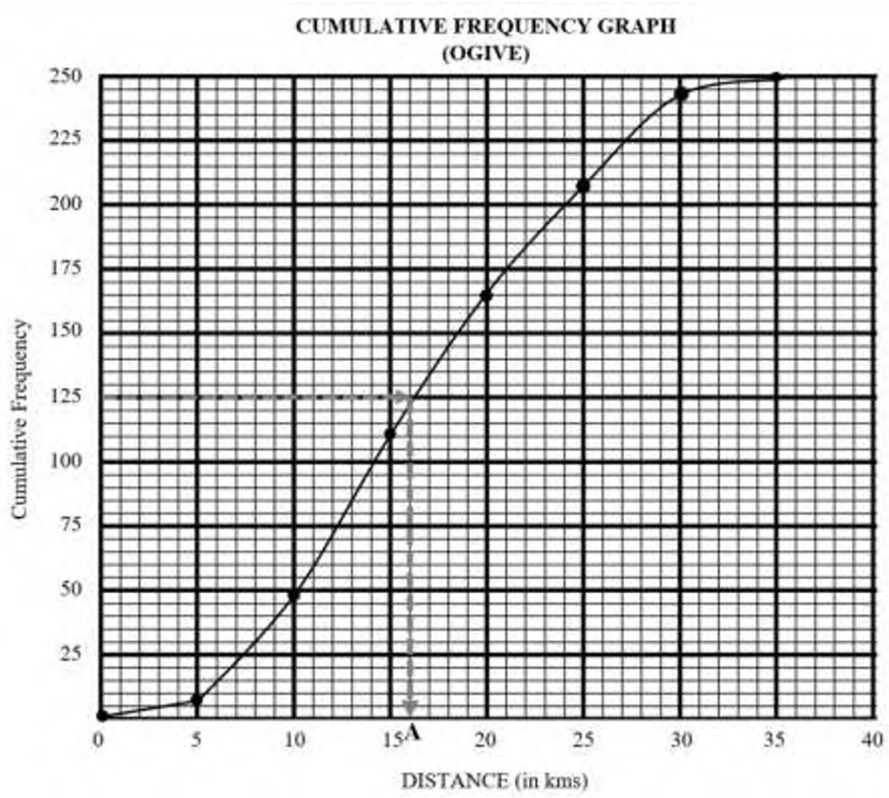
A ✓ 8 and 49

CA ✓ 112 and 164

CA ✓ 205 , 243 and 250

(3)

2.2



A✓ grounded at (0 ; 0)

CA✓ cumulative frequencies for y-coords

CA✓ 5 other points correct

A✓ smooth shape

(4)

2.3

See ogive for dotted lines and point marked A.
Median = 16 km

OR
Answer only: Full marks (accept 15,16,17)

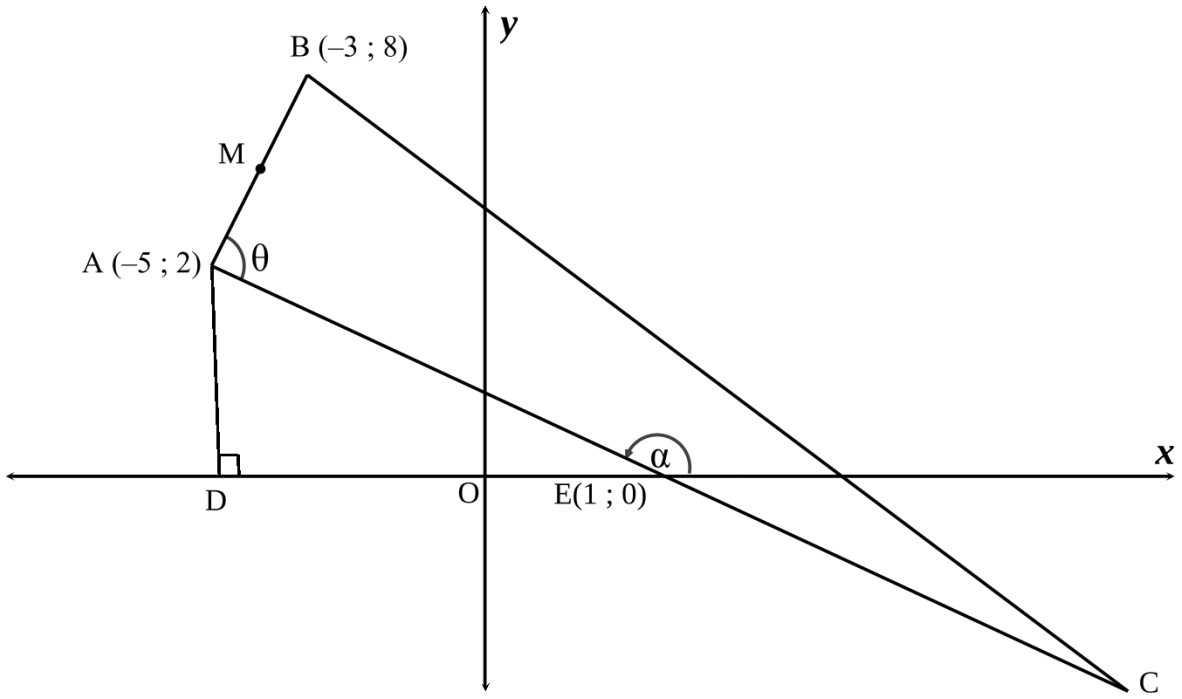
CA✓ indication on graph

CA✓ approx. median

(2)

[9]

QUESTION 3



3.1	$M\left(\frac{x_2+x_1}{2};\frac{y_2+y_1}{2}\right)$ $M\left(\frac{-5-3}{2};\frac{2+8}{2}\right)$ $M(-4;5)$ Answer only: full marks	A✓ Substitution of A and B into midpoint formula CA✓ answer	(2)
3.2	$D(-5;0)$	A✓ answer	(1)
3.3	$\frac{-5+x_C}{2}=1$ $\quad$ $\frac{2+y_C}{2}=0$ $\therefore x_C=7$ $\quad$ $y_C=-2$ $C(7;-2)$ **OR** $C(1+6\,;\,0-2)$ $\quad$ Using transformations $C(7\,;-2)$	A✓ midpoint ITO *x* A✓ midpoint ITO *y* A✓ *x*-value A✓ *y*-value	(2) (2)
3.4	$AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $AC=\sqrt{(-5-7)^2+(2+2)^2}$ $AC=4\sqrt{10}$	A✓ substitute A and C into distance formula CA✓ answer	(2)

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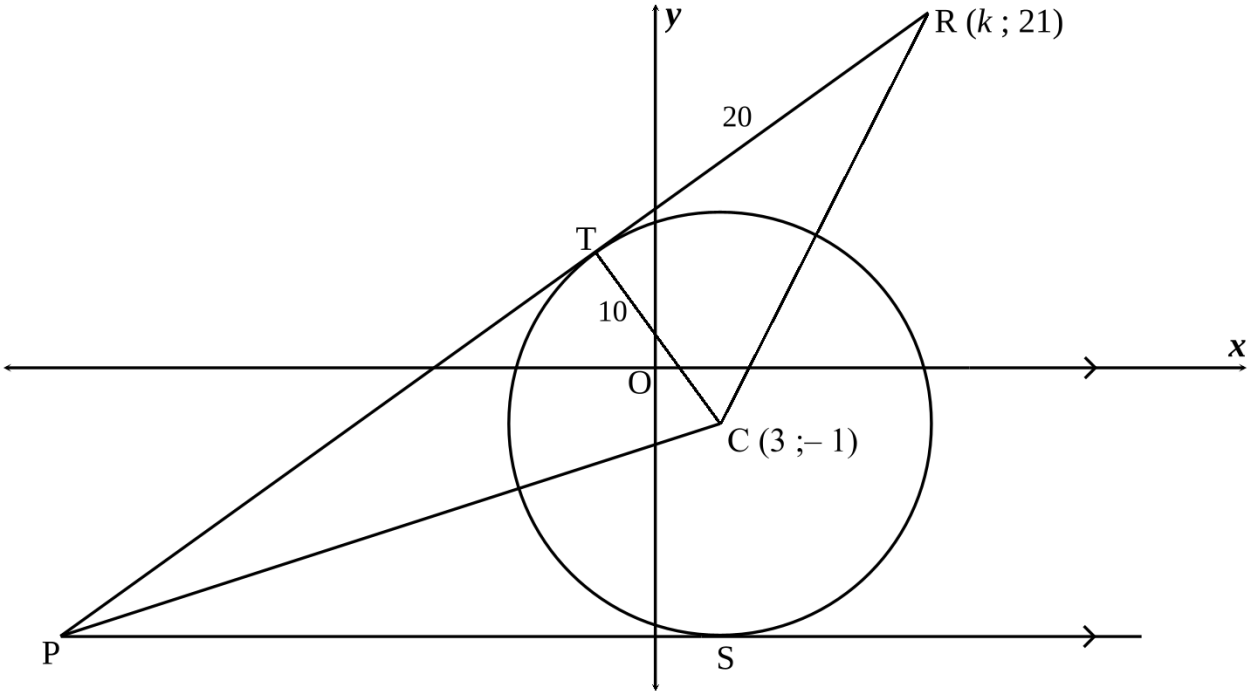
3.5	Method: translation A → B: $(x\ ;\ y) \rightarrow (x+2;\ y+6)$ ∴ by symmetry: C → F: C(7 ; –2) → F(7 + 2 ; –2 + 6) ∴ F(9 ; 4) **OR** Midpoint of intersection of diagonals = $T(2;3)$ Let coordinates of F be $(a\ ;\ b)$ $\frac{a-5}{2}=2 \quad and \quad \frac{b+2}{2}=3$ $a=9 \quad and\ b=4$ ∴ F(9 ; 4)	A✓ *x*-coordinate A✓ *y*-coordinate **OR** A✓ *x*-coordinate A✓ *y*-coordinate	(2) (2)
3.6	$m\text{AB}=\frac{8-2}{-3+5}=3$ ∴ gradient of perpendicular bisector: $-\frac{1}{3}$ $\therefore y=-\frac{1}{3}x+c$ $sub\ (-4;5): \quad 5=-\frac{1}{3}(-4)+c$ $c=\frac{11}{3}$ ∴ equation of perpendicular bisector: $y=-\frac{1}{3}x+\frac{11}{3}$	A✓ $m\text{AB} = 3$ CA✓ $m_{\text{perp bisector}} = -\frac{1}{3}$ CA✓ substitution CA✓ equation	(4)
3.7	$m\text{AC}=\frac{2+2}{-5-7}=-\frac{1}{3}$ $\tan\alpha=-\frac{1}{3}$ $\alpha=\tan^{-1}\left(-\frac{1}{3}\right)+180°$ $\alpha=161,57°$ $\therefore \alpha\approx 162°$ OR Accept method using Cosine Rule	A✓ $m\text{AC}$ CA✓ $\tan\alpha=-\frac{1}{3}$ CA✓ answer	(3)
3.8	$m\text{AB} = 3$ ∴ gradient of new line = 3 (*// lines = gradients*) y = 3x + c sub E(1;0) : 0 = 3(1) + c = –3 y = 3x – 3	CA✓ equal gradients CA✓ answer	(2)

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3.9	$mAB \times mAC = 3 \times -\frac{1}{3}$ $\therefore mAB \times mAC = -1$ $\therefore AB \perp AC$ $\therefore \theta = 90°$ Accept methods using Cosine Rule & Trig ratios	A✓ $mAB \times mAC = -1$ A✓ answer	(2)
3.10	$AC = 4\sqrt{10}$ units $AB = \sqrt{(-3+5)^2 + (8-2)^2} = 2\sqrt{10}$ units $\therefore$ Area of $\Delta ABC = \frac{1}{2}AC.AB$ $\therefore$ Area of $\Delta ABC = \frac{1}{2}\left(4\sqrt{10}\right)\left(2\sqrt{10}\right)$ $\therefore$ Area of $\Delta ABC = 40$ units^2	A✓ substitute A and B into distance formula CA✓ length of AB CA✓ substitution CA✓ answer	(4)
			[24]

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QUESTION 4

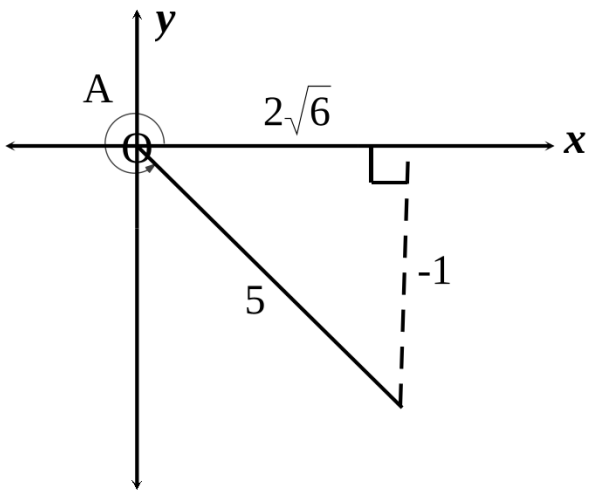


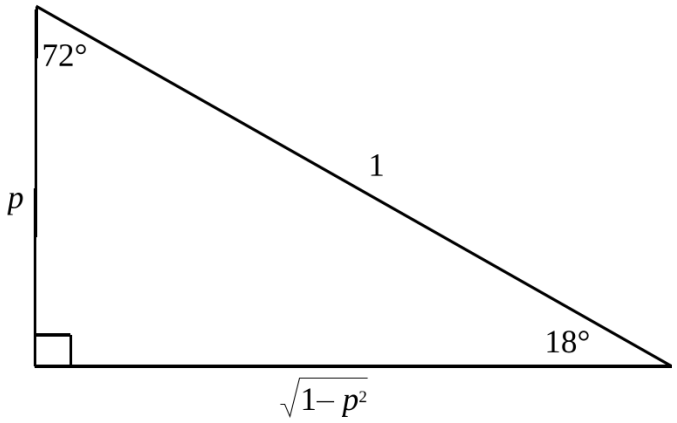
4.1	Radius is perpendicular to tangent	A✓ reason	(1)
4.2	$CR = \sqrt{10^2 + 20^2} = \sqrt{500}$ (pythag) $CR = \sqrt{(k-3)^2 + (21+1)^2} = \sqrt{(k-3)^2 + 484}$ $\therefore \sqrt{(k-3)^2 + 484} = \sqrt{500}$ $\therefore (k-3)^2 + 484 = 500$ $\therefore (k-3)^2 = 16$ $\therefore k - 3 = \pm 4$ $\therefore k = -1$ or $k = 7$ but $k \neq -1$ (given that R is the first quadrant) $k = 7$ only	A✓ 500 A✓ equate CA✓ simplification CA✓ both values and rejection of $k = -1$	(4)
4.3	$(x-3)^2 + (y+1)^2 = 100$	A✓ $(x-3)^2 + (y+1)^2$ A✓ 100	(2)
4.4	$S(3;-11)$ $\therefore y = -11$	A✓ answer	(1)
4.5.1	$y = -11$ *Eq* 1 $3y = 4x + 35$ *Eq* 2 Sub *Eq* 1 into *Eq* 2: $3(-11) = 4x + 35$ $4x = -68$ $x = -17$ $\therefore P(-17;-11)$	CA✓ substitution CA✓ x – value	(2)

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4.5.2	$PC=\sqrt{(3+17)^2+(-1+11)^2}=10\sqrt{5}$ $TC=10$ $PT^2=PC^2-TC^2$ (pythag) $PT=\sqrt{\left(10\sqrt{5}\right)^2-(10)^2}$ $PT=20$ units **OR** $PT=PS$ (tangents from common point) $PS=3-(-17)=20$ units $\therefore PT=20$ units	CA✓ $PC^2=500$ $PC=10\sqrt{5}$ CA✓ Pythagoras CA✓ answer A✓ S/R CA✓ Substitution CA✓ answer	(3) (3)
			[13]

QUESTION 5

5.1.1	 $y=\sqrt{r^2-x^2}$ $y=\sqrt{(5)^2-\left(2\sqrt{6}\right)^2}=1$ $\therefore y=-1$ (quadrant 4) $\therefore \quad -\sqrt{6}.\tan A$ $=-\sqrt{6}\times\left(\frac{-1}{2\sqrt{6}}\right)$ $=\frac{1}{2}$	A✓ diagram A✓ $y=-1$ CA✓ $\tan A=\frac{2\sqrt{6}}{-1}$ CA✓ answer	(4)

5.1.2	$\sin 2A = 2\sin A.\cos A$ $= 2\left(\frac{-1}{5}\right)\left(\frac{2\sqrt{6}}{5}\right)$ $= -\frac{4\sqrt{6}}{25}$	A✓ double angle identity CA✓ $\sin A=\frac{-1}{5}$ CA✓ $\cos A=\frac{2\sqrt{6}}{5}$ CA✓ answer	(4)
5.2			
5.2.1	$\cos 18° = \sqrt{1-p^2}$	A✓ diagram A✓ answer Answer only full marks	(2)
5.2.2	$\cos 48° = \cos(30°+18°)$ $= \cos 30°\cos 18° - \sin 30°\sin 18°$ $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{1-p^2}}{1}\right) - \left(\frac{1}{2}\right)\left(\frac{p}{1}\right)$ $= \frac{\sqrt{3}.\sqrt{1-p^2} - p}{2}$ $= \frac{\sqrt{3-3p^2} - p}{2}$	A✓ $(30°+18°)$ A✓ expansion A✓ special angle substitution CACA✓✓substitution of $\sin 18°$ and $\cos 18°$	(5)
5.2.3	$\cos 18° = 1-2\sin^2 9°$ $-2\sin^2 9° = \cos 18° - 1$ $\sin^2 9° = \frac{\cos 18° - 1}{-2}$ $\sin 9° = \sqrt{\frac{\cos 18° - 1}{-2}}$ $\sin 9° = \sqrt{\frac{\sqrt{1-p^2} - 1}{-2}} = \sqrt{\frac{1-\sqrt{1-p^2}}{2}}$	A✓ double angle expansion A✓ making $\sin 9°$ the subject CA✓ answer	(3)
			[18]

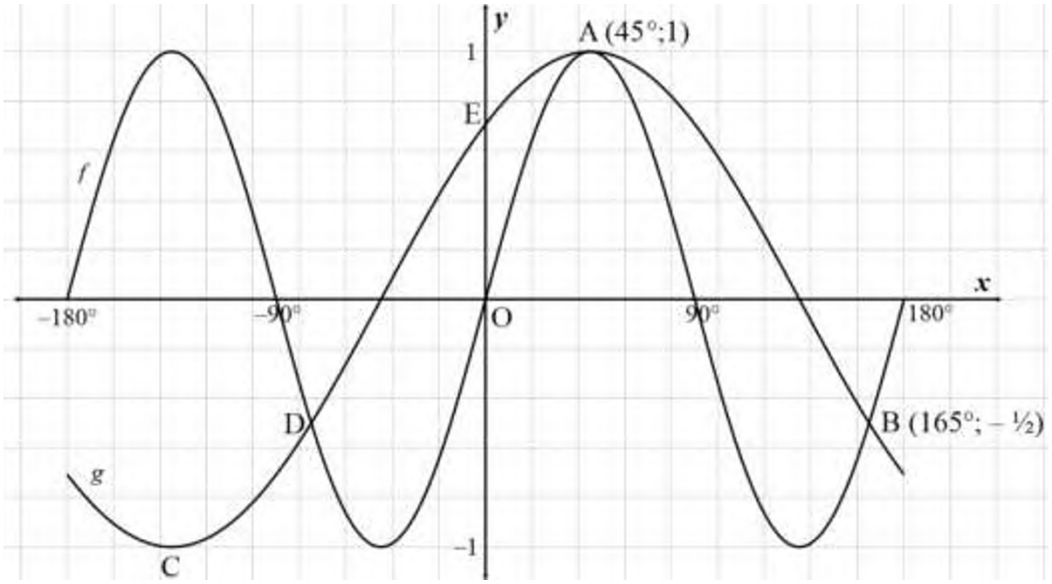
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QUESTION 6

6.1	$\frac{\sin(180°-x).\tan(x-180°).\cos(360°+x)}{\sin^2(180°+x)+\sin^2(90°-x)}$ $=\frac{\sin x.\tan x.\cos x}{\sin^2 x+\cos^2 x}$ $=\sin x.\frac{\sin x}{\cos x}.\cos x$ $=\sin^2 x$	A✓ $\sin x$ A✓ $\tan x$ A✓ $\sin^2 x$ A✓ $\cos^2 x$ A✓ $\tan x=\frac{\sin x}{\cos x}$ CA✓ answer	(6)
6.2	$\frac{\cos 330°.\tan 150°.\sin 12°}{\tan 675°.\cos 258°}$ $=\frac{(\cos 30°).(-\tan 30°).(\sin 12°)}{(-\tan 45°).(-\cos 78°)}$ $=-\frac{(\cos 30°).(\tan 30°).(\sin 12°)}{(\tan 45°).(\sin 12°)}$ $=-\frac{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{3}\right)}{1}$ $=-\frac{1}{2}$	A✓ $\cos 30°$ A✓ $-\tan 30°$ A✓ co ratio A✓ $-\cos 78°$ A✓ $\tan 45°$ A✓ special angles CA✓ answer	(7)
6.3.1	LHS: $\frac{\cos\alpha+\cos 2\alpha}{\sin 2\alpha-\sin\alpha}$ $=\frac{\cos\alpha+(2\cos^2\alpha-1)}{(2\sin\alpha\cos\alpha)-\sin\alpha}$ $=\frac{2\cos^2\alpha+\cos\alpha-1}{2\sin\alpha\cos\alpha-\sin\alpha}$ $=\frac{(2\cos\alpha-1)(\cos\alpha+1)}{\sin\alpha(2\cos\alpha-1)}$ $=\frac{(\cos\alpha+1)}{\sin\alpha}$ = RHS	A✓ cos double angle A✓ sin double angle A✓ numerator factors A✓ denominator factors	(4)
6.3.2	$\sin 2\alpha-\sin\alpha=0$ $2\sin\alpha\cos\alpha-\sin\alpha=0$ $\sin\alpha(2\cos\alpha-1)=0$ $\therefore \sin\alpha=0$ or $2\cos\alpha-1=0$ $\cos\alpha=\frac{1}{2}$ $\alpha=0°+k.180;k\in Z$ $\alpha=\pm 60°+k.360;k\in Z$ N.B. If $\sin\alpha=0$ is solved only – max. 3/5 marks	A✓ equating to 0 A✓ factors CA✓ $0°+k.180$ CA✓ $\pm 60°+k.360$ A✓ $k\in Z$	(5)
			[22]

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QUESTION 7



7.1	$a = -45°$	A✓ answer	(1)
7.2	$180°$	A✓ answer	(1)
7.3	$C(-135°;-1)$ E: $y = \cos(0° + 45°)$ $y = \frac{\sqrt{2}}{2}$ $E\left(0;\frac{\sqrt{2}}{2}\right)$	A✓ $C(-135°;-1)$ A✓ solving for y A✓ $E\left(0;\frac{\sqrt{2}}{2}\right)$	(3)
7.4	Amplitude = 3	A✓ answer	(1)
7.5.1	$x \in [0^\circ;165^\circ); x \neq 45°$ OR $x \in [0^\circ;45^\circ) \cup (45°;165°)$ OR $0° \leq x < 165°; x \neq 45°$	A✓ $[0^\circ;165^\circ)$ A✓ $x \neq 45°$ A✓ critical values A✓ notation A✓ $0° \leq x < 165°$ A✓ $x \neq 45°$	(2)
7.5.2	$0° \leq x \leq 135°$	A✓ critical values A✓ notation	(2)
7.6	$\sqrt{2}\sin 2x = \cos x + \sin x$ $\sin 2x = \frac{\cos x}{\sqrt{2}} + \frac{\sin x}{\sqrt{2}}$ $\sin 2x = \cos x \frac{1}{\sqrt{2}} + \sin x \frac{1}{\sqrt{2}}$ $\sin 2x = \cos x.\cos 45° + \sin x.\sin 45°$ $\sin 2x = \cos(x - 45°)$	A✓ division of $\sqrt{2}$ A✓ identifying compound $\angle$ $\cos(x - 45°)$	

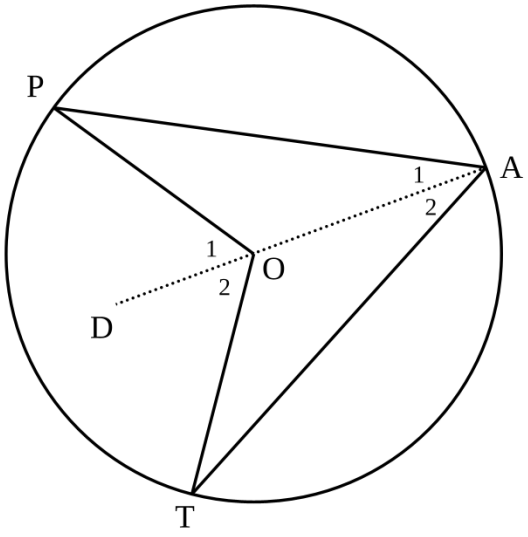
	Equating the 2 functions gives the points of intersection of the 2 graphs.	A✓ conclusion	(3)
			[13]



QUESTION 8

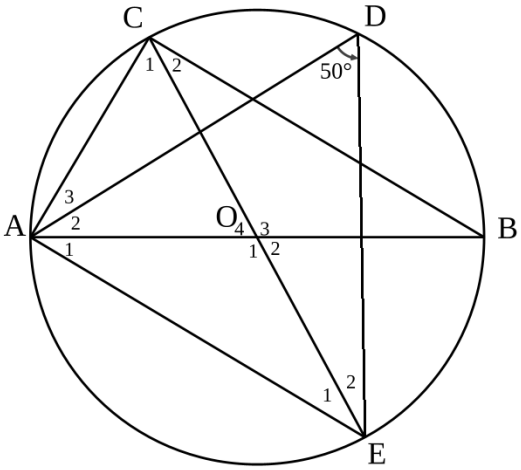
8.1	bisects the chord.	A✓ answer	(1)
8.2.1	$AM = 4\,cm$ (line from centre $\perp$ chord)	A✓ S A✓R	(2)
8.2.2	$OM = (r-2)$ $\therefore \quad r^2 = (r-2)^2 + 4^2$ (pythag) $r^2 = r^2 - 4r + 4 + 16$ $4r = 20$ $r = 5$ cm	A✓ $OM = (r-2)$ CA✓ $r^2 = (r-2)^2 + 4^2$ CA✓ simplification CA✓ answer	(4)
			[7]

QUESTION 9



9.1	Constr: Draw line AO and extend to D. Proof: $OP = OT$ (radii) $\therefore \hat{A}_1 = \hat{P}$ ($\angle$s opp = sides) but $\hat{O}_1 = \hat{A}_1 + \hat{P}$ (ext $\angle$ of Δ) $\therefore \hat{O}_1 = 2\hat{A}_1$ *Similarly* $\hat{O}_2 = 2\hat{A}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\left(\hat{A}_1 + \hat{A}_2\right)$ $\therefore P\hat{O}T = 2P\hat{A}T$	A✓ construction A✓ S/R A✓ S/R A✓ S A✓ S	(5)

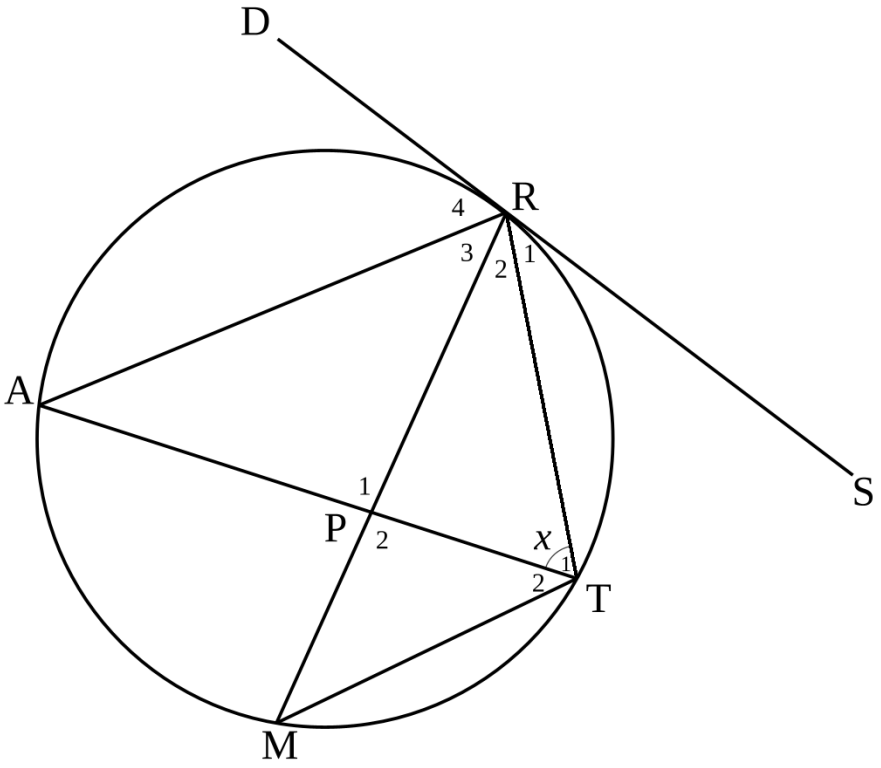
9.2 **Do not mark this question!!! (Maximum for Paper 2 – 141)**



9.2.1a	$\hat{O}_1 = 100°$	($\angle$ at centre $= 2 \times \angle$ at circ)	A✓ S A✓R	(2)
9.2.1b	$\hat{O}_1 = 100°$ $\hat{A}_1 = \hat{E}_1$ $\therefore \hat{E}_1 = \frac{180° - 100°}{2}$ $\therefore \hat{E}_1 = 40°$	($\angle$s opp = radii) (sum $\angle$s Δ)	A✓ S A✓R CA✓ S/R	(3)
9.2.2	$\hat{A}_1 = \hat{E}_1$ but $\hat{A}_1 = \hat{C}_2$ $\therefore \hat{E}_1 = \hat{C}_2$ $\therefore AE \parallel CB$ OR $\hat{A}_1 = \hat{E}_1$ but $\hat{E}_1 = \hat{B}$ $\therefore \hat{A}_1 = \hat{B}$ $\therefore AE \parallel CB$ OR	($\angle s$ opp = radii) ($\angle s$ in same segment) (alt $\angle s$ =) ($\angle s$ opp = radii) ($\angle s$ in same segment) (alt $\angle s$ =)	A✓ S/R A✓ S/R A✓ S A✓ R A✓ S/R A✓ S/R A✓ S A✓ R A✓ S/R A✓ S/R	(4) (4)

	$C\hat{A}E = 90°$ ($\angle s$ in semi $\square$) $A\hat{C}B = 90°$ ($\angle s$ in semi $\square$) $C\hat{A}E + A\hat{C}B = 180°$ $\therefore AE \square CB$ (co - int $\angle s$)	A✓ S A✓ R	(4)
			[14]

QUESTION 10

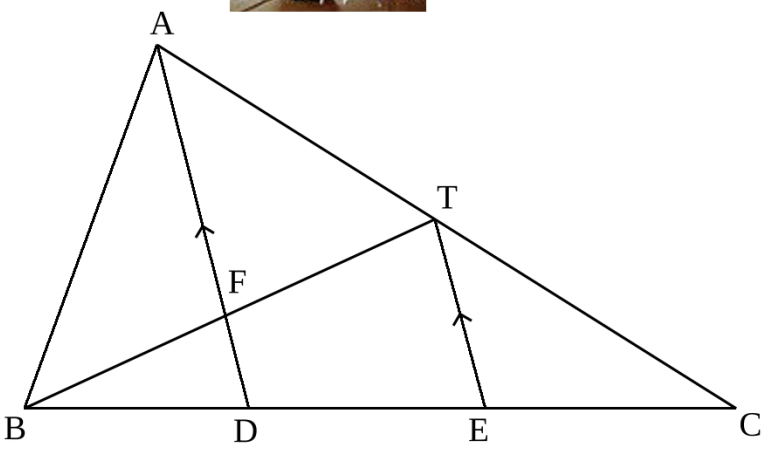


10.1.1	$\hat{R}_3 = \hat{T}_2$ ($\angle s$ in same segment) $\hat{R}_4 = \hat{T}_1$ (tan chord theorem) but $\hat{T}_1 = \hat{T}_2$ (given AT bisects MTR) $\therefore \hat{R}_3 = \hat{R}_4$	A✓ S A✓R A✓ S A✓R	(4)
10.1.2	In ΔAPR and ΔMPT 1. $\hat{A} = \hat{M}$ ($\angle s$ in same segment) 2. $\hat{P}_1 = \hat{P}_2$ (vert opp $\angle s$) 3. $\hat{R}_3 = \hat{T}_2$ (remaining $\angle s$ Δ) $\therefore$ $\Delta APR \,|||\, \Delta MPT$ ($\angle\angle\angle$)	A✓ S/R A✓ S/R A✓ R	(3)

10.2	$\frac{AR}{MT}=\frac{PR}{PT}$ ($\|\|\|\ \Delta s$) $AR=\frac{3}{2}MT$ (*given*) $\therefore \frac{AR}{MT}=\frac{3}{2}$ $\therefore \frac{PT}{PR}=\frac{2}{3}$	A✓ S A✓R A✓ S	(3)
			[10]

QUESTION 11





11.1	$\frac{CE}{ED}=\frac{1}{2}$	A✓ S	(1)
11.2	$DE=\frac{2}{3}(9\,cm)=6\,cm$	A✓ S	(1)
11.3	$\frac{TE}{2}=\frac{BE}{BD}=\frac{2}{1}$(/// triangles) $\therefore TE=4cm$ **Accept solution using the Midpoint theorem**	A✓ S/R A✓ S	(2)
11.4.1	$\frac{\text{Area of }\Delta ADC}{\text{Area of }\Delta ABD}=\frac{\frac{1}{2}DC\times ⍰}{\frac{1}{2}BD\times ⍰}=\frac{6cm}{6cm}=1$	A✓Areas A✓ Answer	(2)

11.4.2	$\frac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC}$ $= \frac{\text{Area of } \Delta TEC}{\text{Area } \Delta TBC} \times \frac{\text{Area of } \Delta TBC}{\text{Area } \Delta ABC}$ $= \frac{EC}{BC} \times \frac{TC}{AC}$ $= \frac{1}{4} \times \frac{1}{3}$ $= \frac{1}{12}$ **OR** $\text{Area of } \Delta TEC = \frac{1}{4}(\text{Area of } \Delta TBC)$ (common vertex = altit.) $= \frac{1}{4}\left(\frac{1}{3}\text{Area of } \Delta ABC\right)$ (common vertex = altit.) $\frac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC} = \frac{1}{12}$ OR **Accept area rule with the use of the angle of $\widehat{C}$.**	A✓ S A✓ S A✓Answer A✓ S A✓ S A✓ S	(3) (3)
			[9]
		TOTAL:	**150**

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