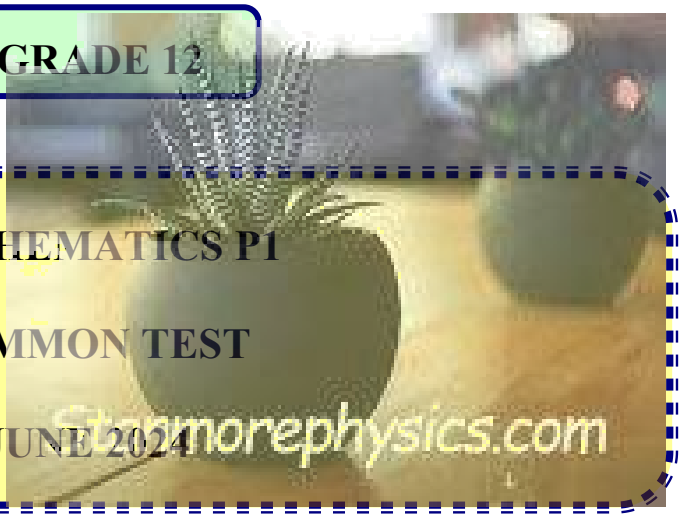
KWAZULU-NATAL PROVINCE

EDUCATION

REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12



MATHEMATICS P1

COMMON TEST

JUNE 2024

MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages and an information sheet.

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.

5. Answers only will NOT necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.

8. Write neatly and legibly.

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QUESTION 1

1.1 Solve for x:

1.1.1 $5x(2x+7)(8-x)=0$ (3)

1.1.2 $x^2+13x+12=0$ (3)

1.1.3 $5x^2-7x+8=0$ (2)

1.1.4 $\sqrt{x-2}+2=x$ (5)

1.1.5 $x(x-1)<20$ (4)

1.1.6 $2^{1-2x}+7.2^{-x}-4=0$ (5)

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1.2 The roots of a quadratic equation are $x=\dfrac{5\pm\sqrt{22-3m}}{2}$.

If m is an integer, determine the largest value of m for which these roots will be rational. (3)

1.3 Evaluate: $\dfrac{\sqrt{9^{2024}}}{\sqrt{9^{2023}}-\sqrt{9^{2025}}}$ (3)

1.4 Solve simultaneously for x and y:

$3+y-2x=0$ and $4x^2+y^2-2xy-7=0$ (6)

[34]

QUESTION 2

Given the quadratic sequence: -5 ; 12 ; 27 . . . .

2.1 Determine the next two terms of the quadratic sequence. (2)

2.2 Determine the expression for the general term of the quadratic sequence. (4)

2.3 Which term(s) of the quadratic sequence has a value of 51? (3)

2.4 Calculate the value of $\sum_{n=3}^{10} T_n-\sum_{n=11}^{17} T_n$. (3)

[12]

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QUESTION 3

3.1 Consider the arithmetic sequence: $-\frac{7}{2}\ ;\ -3\ ;\ -\frac{5}{2}\ldots.$

3.1.1 Determine the general term of the sequence. (2)

3.1.2 The sum of the first n terms of this sequence is 675.
Calculate the value of n. (4)

3.1.3 A new sequence is formed by squaring each term of the given arithmetic sequence. Determine which term of the new sequence will have the smallest value. (3)

3.2 The first 3 terms of an infinite geometric series are given:
$(x+1)+2(x+1)^2+4(x+1)^3+.....$

3.2.1 For which values of x will the series converge? (3)

3.2.2 If $x=-\frac{3}{4}$, determine the numerical value of the first term. (1)

3.2.3 Write the series in sigma notation. (3)

3.2.4 Calculate the sum to infinity of the series. (2)

[18]

QUESTION 4

It is given that the asymptotes of $f(x)=\frac{6}{x+p}+q$ intersect at $(4\ ;\ 3)$.

4.1 Write down the equation of f. (2)

4.2 Determine the intercepts of f with the axes. (3)

4.3 Sketch the graph of f, clearly showing all the intercepts with the axes and any asymptotes. (3)

4.4 g is one of the axes of symmetry of f and it is a decreasing function. Determine the equation of g. (3)

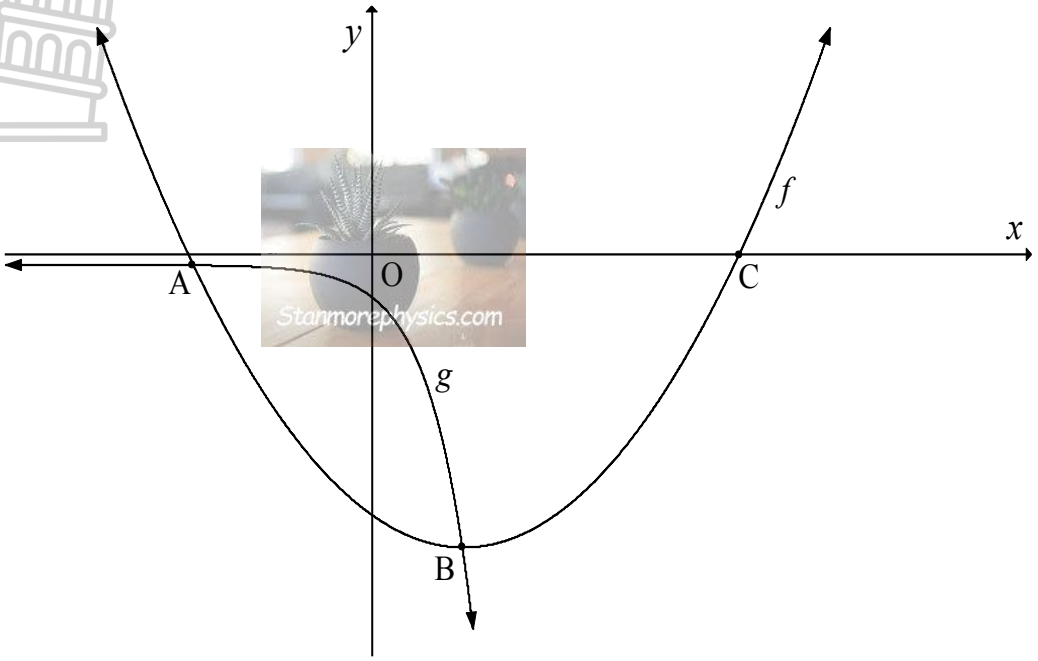
4.5 $(-3\ ;\ 2)$ is a point on f. Determine the coordinates of the image of this point after reflection in g. (2)

[13]

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QUESTION 5

The graphs of $f(x)=(x-1)^2-9$ and $g(x)=-a^x$ are drawn below. The graph of g cuts f at points A and B. B is the turning point of f.



5.1 Write down the coordinates of B. (2)

5.2 For which values of x are both graphs decreasing? (1)

5.3 Determine the coordinates of the x-intercepts of f. (3)

5.4 Show that $a=9$. (1)

5.5 Determine the equation of g^{-1} in the form $y=.....$ (2)

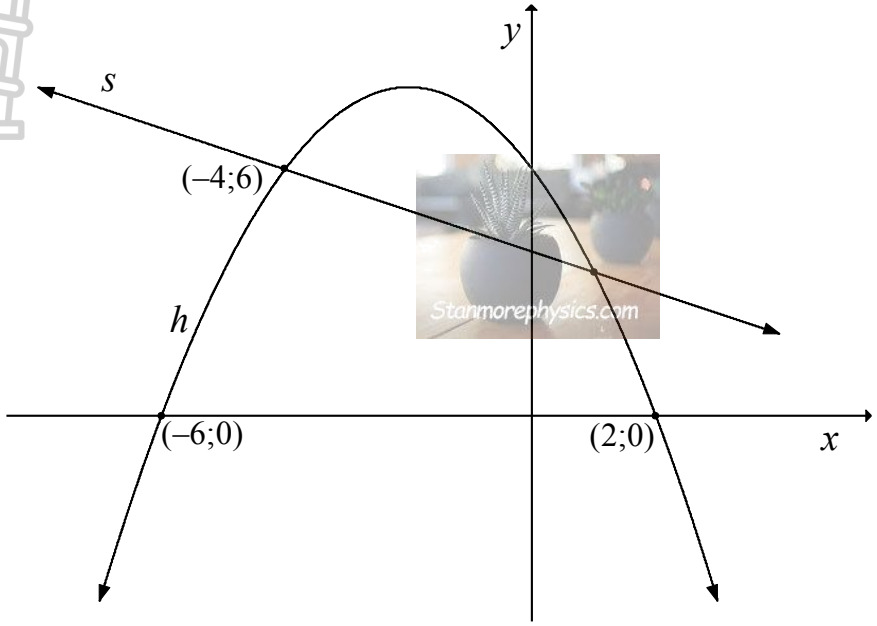
5.6 Sketch the graph of g^{-1}, indicating any intercepts with the axes. (2)

5.7 For which values of x is $g^{-1}(x)>2$? (3)

[14]

QUESTION 6

The graphs of $h(x) = ax^2 + bx + c$ and $s(x) = mx + c$ are drawn below.
The x-intercepts of h are $(-6\ ;\ 0)$ and $(2\ ;\ 0)$.
$(-4\ ;\ 6)$ are the coordinates of one of the points of intersection between h and s.



6.1 Show that $a = -\frac{1}{2}$, $b = -2$ and $c = 6$. (4)

6.2 Determine the maximum value of $h(x)$. (3)

6.3 Determine the equation of s, if it is given that the gradient of s is equal to $-\frac{1}{2}$. (2)

6.4 For which values of k will $s(x) + k = h(x)$ have two real roots that are opposite in sign? (2)

6.5 Describe the translation that h will undergo to become p, where $p(x) = -\frac{1}{2}(x + 2)^2$. (2)

[13]

QUESTION 7

7.1 Given: $f(x) = 2x^2 + 4$

7.1.1 Determine the derivative of f from first principles. (5)

7.1.2 A tangent to the graph of f has a gradient of -12. Determine the equation of the tangent. (4)

7.2 Determine the following:

7.2.1 $f'(x)$ if $f(x) = \dfrac{2x^2 - 5x - 12}{x - 4}$ (3)

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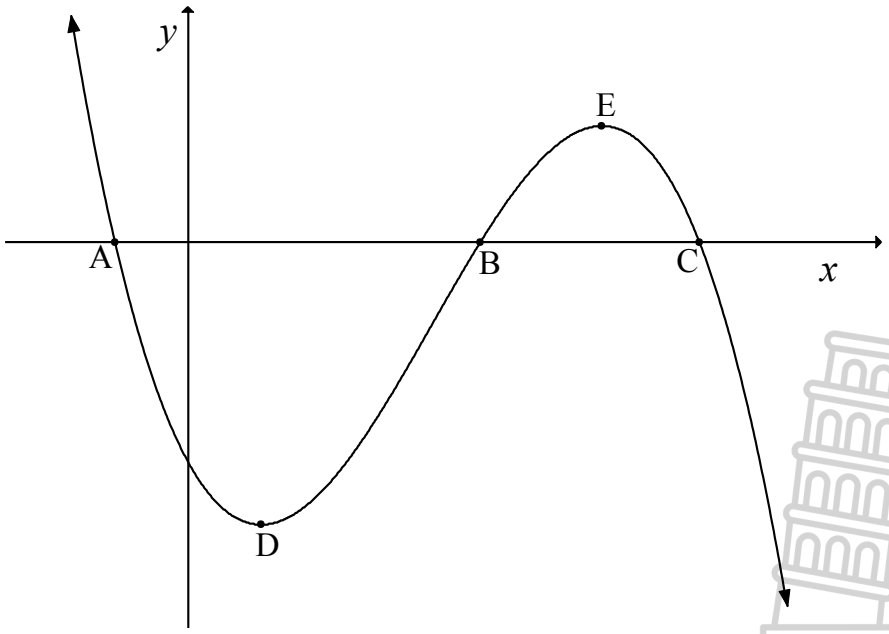
7.2.2 $\mathrm{D}_x\left[\sqrt[5]{x^2} + x(x-9)\right]$ (4)

7.2.3 $\dfrac{dy}{dx}$ if $y = \dfrac{x}{6} - \dfrac{6}{x}$ (3)

[19]

QUESTION 8

The diagram shows the graph of $f(x) = -x^3 + 10x^2 - 17x - 28$.
A, B and C are the x-intercepts of the graph, and D and E the turning points.



8.1 Calculate the coordinates of A, B and C. (4)

8.2 Calculate the coordinates of D and E. (4)

8.3 Determine the values of x for which

8.3.1 the graph is concave down. (3)

8.3.2 $f'(x)$ is increasing. (2)

[13]

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QUESTION 9

Given: A cubic function f with the following properties.

- The x-intercepts of the graph of $f'(x)$ are -2 and 4.
- $f''(x) > 0$ for $x > 1$
- The graph of f has only one x-intercept.
- $f(0) > 0$

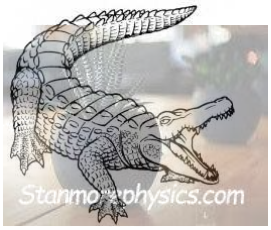
Use the given information to draw a sketch graph of f.
It is not necessary to indicate the values of the x- or y-intercepts of the graph, but only the x- coordinates of the turning points. (5)

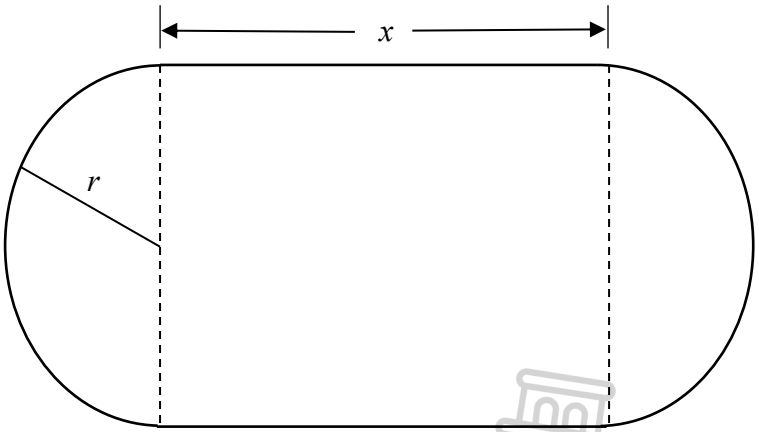
[5]

QUESTION 10

The managers of a zoo are planning to build a fence around a crocodile enclosure.
The sketch below shows the shape of the enclosure.
The length of the straight sections will be x meters each, and the radius of the semi-circular end sections r meters each, as shown in the sketch.
The total area of the enclosure will be 400 m^2.

.





10.1 Show that $x = \dfrac{400 - \pi r^2}{2r}$ (2)

10.2 Show that the length of fencing required (L) can be expressed as $L(r) = \dfrac{400}{r} + \pi r$. (3)

10.3 Calculate the value of the radius that will ensure that the length of fencing required will be a minimum, so as to minimise the cost of building the fence. (4)

[9]

TOTAL: 150

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INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

InΔABC: $\dfrac{a}{\sin \mathrm{A}} = \dfrac{b}{\sin \mathrm{B}} = \dfrac{c}{\sin \mathrm{C}}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta\mathrm{ABC} = \frac{1}{2}ab.\sin \mathrm{C}$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad \mathrm{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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