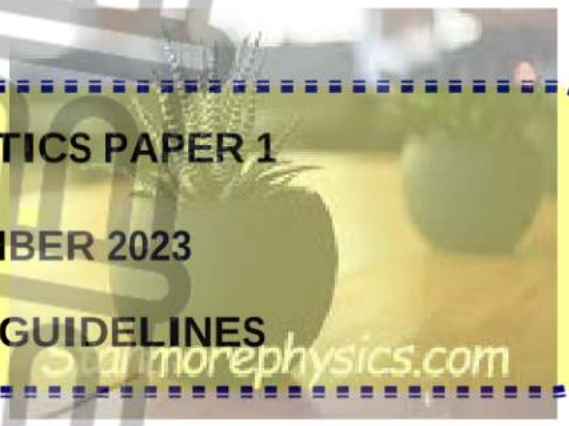
KWAZULU-NATAL PROVINCE

EDUCATION

REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11



MATHEMATICS PAPER 1

NOVEMBER 2023

MARKING GUIDELINES

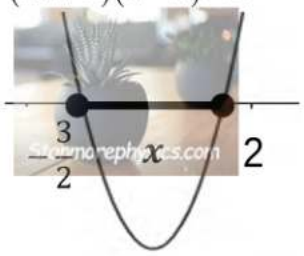
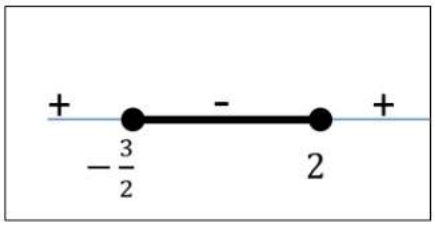
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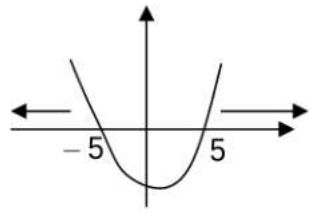
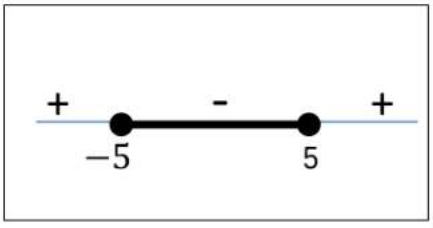
Stanmorephysics

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QUESTION 1

1.1.1	$(2x+1)(5-x)=0$ $x=-\frac{1}{2}$ or $x=5$	✓ answer ✓ answer (2)
1.1.2	$x^2-\frac{2}{3}x-5=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-\left(-\frac{2}{3}\right)\pm\sqrt{\left(-\frac{2}{3}\right)^2-4(1)(-5)}}{2(1)}$ $x=2,59$ or $x=-1,93$	✓ substitution into correct formula ✓ answer ✓ answer (3)
1.1.3	$2x\left(x-\frac{1}{2}\right)\leq 6$ $2x^2-1x-6\leq 0$ $(2x+3)(x-2)\leq 0$   $\frac{-3}{2}\leq x\leq 2$ **OR** $x\in\left[-1\frac{1}{2};2\right]$	✓ standard form ✓ factors ✓ ✓ $\frac{-3}{2}\leq x\leq 2$ (4)
1.1.4	$x+\sqrt{2x-3}=3$ $\sqrt{2x-3}=3-x$ $\left(\sqrt{2x-3}\right)^2=(3-x)^2$ $2x-3=9-6x+x^2$ $x^2-8x+12=0$ $(x-6)(x-2)=0$ $x\neq 6$ or $x=2$	✓ isolating the root ✓ squaring both sides ✓ standard form ✓ $x\neq 6$ ✓ $x=2$ (5)

1.2	$3x-y=1$ and $2xy-3y^2=-x^2-7$ $y=3x-1$.....(3) $2x(3x-1)-3(3x-1)^2=-x^2-7$ $6x^2-2x-27x^2+18x-3+x^2+7=0$ $-20x^2+16x+4=0$ $5x^2-4x-1=0$ $(5x+1)(x-1)=0$ $x=\frac{-1}{5}$ or $x=1$ $y=3\left(\frac{-1}{5}\right)-1$ $y=3(1)-1$ $y=\frac{-8}{5}$ or $y=2$	equation 1 & equation 2 resp. ✓ equation 3 ✓ substitution into equation 2 ✓ standard form ✓ factors ✓ both x-values ✓ both y-values (6)
1.3	$x=\frac{-10\pm\sqrt{100-4k^2}}{2k}$	
1.3.1	$100-4k^2=0$ $-4(k^2-25)=0$ $(k+5)(k-5)=0$ $k=5$ or $k=-5$ **OR** $100-4k^2=0$ $4k^2=100$ $k^2=25$ $k=\pm5$	✓ $100-4k^2=0$ ✓ factors ✓ both answers for k (3) **OR** ✓ $100-4k^2=0$ ✓ isolating the square ✓ both answers for k (3)
1.3.2	$100-4k^2<0$ $(k+5)(k-5)>0$ CV: $k=-5$ or $k=5$   $k<-5$ or $k>5$ **OR** $k\in(-\infty;-5)\ \cup\ (5;\infty)$	✓ $100-4k^2<0$ ✓✓ $k<-5$ or $k>5$ (3)
		[26]

QUESTION 2

2.1.1	$\frac{\left(9x^2\right)^{-\frac{1}{2}}}{9x^{-2}}$ $= \frac{3^{2\left(\frac{-1}{2}\right)} \cdot x^{2\left(\frac{-1}{2}\right)}}{3^2 \cdot x^{-2}}$ $= \frac{3^{-1} \cdot x^{-1}}{3^2 \cdot x^{-2}}$ $= 3^{-1-2} \cdot x^{-1+2}$ $= 3^{-3} \cdot x^1$ $= \frac{1}{27}x$ **OR** $\frac{\left(9x^2\right)^{-\frac{1}{2}}}{9x^{-2}}$ $= \frac{x^2}{9\left(9x^2\right)^{\frac{1}{2}}}$ $= \frac{x^2}{9(3x)}$ $= \frac{x}{27}$ or $\frac{1}{27}x$	✓ splitting to prime bases & removing brackets around bases ✓ applying exponential law ✓ answer (3) **OR** ✓ changing to positive exponents ✓ applying exponential law ✓ answer (3)
2.1.2	$\frac{\sqrt{8k^4} - \sqrt{98k^4}}{\sqrt{25k^2}}$ $= \frac{\sqrt{8} \cdot \sqrt{k^4} - \sqrt{98} \cdot \sqrt{k^4}}{\sqrt{25k^2}}$ $= \frac{\sqrt{4 \times 2} \cdot k^2 - \sqrt{49 \times 2} \cdot k^2}{5k^2}$ $= \frac{2\sqrt{2}k^2 - 7\sqrt{2}k^2}{5k^2}$ $= \frac{-5\sqrt{2}k^2}{5k^2}$ $= -\sqrt{2}$	✓ applying rule: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ to simplify the surds ✓ $\frac{-5\sqrt{2}k^2}{5k^2}$ ✓ answer (3)

2.2.1	$x^{-\frac{3}{2}} = 125$ $x^{-\frac{3}{2}} = 5^3$ $x^{-\frac{3}{2}\left(-\frac{2}{3}\right)} = 5^{3\left(-\frac{2}{3}\right)}$ $x = 5^{-2}$ $x = \frac{1}{25}$	✓ expressing 125 as 5^3 ✓ raising both sides to reciprocal exponent of $-\frac{2}{3}$ ✓ answer (3)
2.2.2	$\frac{p^x . p^{\frac{2}{x}}}{p^{5-x}} = 1$ $p^{x+\frac{2}{x}-5+x} = p^0$ $x + \frac{2}{x} - 5 + x = 0$ $2x^2 - 5x + 2 = 0$ $(2x-1)(x-2) = 0$ $x = \frac{1}{2}$ or $x = 2$ **OR** $\frac{p^x . p^{\frac{2}{x}}}{p^{5-x}} = 1$ $p^{x+\frac{2}{x}} = p^{5-x}$ $x + \frac{2}{x} = 5 - x$ $2x^2 - 5x + 2 = 0$ $(2x-1)(x-2) = 0$ $x = \frac{1}{2}$ or $x = 2$	✓ laws of exponents ✓ equating bases ✓ standard form ✓ both answers (4) **OR** ✓ law of exponents ✓ equating bases ✓ standard form ✓ answers (4)

2.2.3	$\frac{x-4}{\sqrt{x}-2}=5a^0$	
	$\frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2}=5(1)$	✓ $\left(\sqrt{x}+2\right)\left(\sqrt{x}-2\right)$ ✓ $a^0=1$
	$\sqrt{x}+2=5$	
	$\sqrt{x}=3$	✓ $\sqrt{x}=3$
	$x=9$	✓ answer (4)
	OR	**OR**
	$\frac{x-4}{\sqrt{x}-2}=5a^0$	
	$x-4=5(1)(\sqrt{x}-2)$	✓ $a^0=1$
	$x-4=5\sqrt{x}-10$	
	$(x+6)^2=(5\sqrt{x})^2$	✓ isolating the root and squaring both sides
	$x^2+12x+36=25x$	
	$x^2-13x+36=0$	✓ standard form
	$(x-9)(x-4)=0$	
	$x=9$ or $x\neq 4$	✓ answer (4)
		[17]

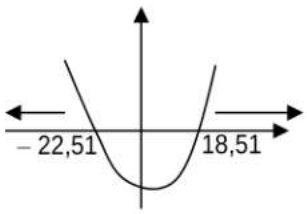
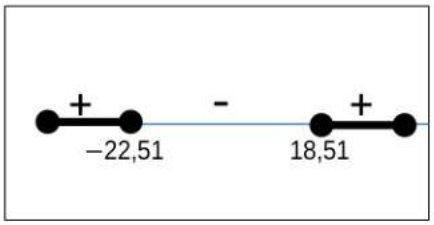
QUESTION 3

Consider the following linear number pattern: $3x+8$; $x-2$; … ; -141 ; -145.

3.1	**finite** linear number pattern	✓ finite (1)
3.2	$3x+8-4=x-2$ $2x=-6$ $x=-3$	✓ $d=-4$ (from last 2 terms) ✓ substitution: $T_1-4=T_2$ (2)
3.3	-1 : -5 ; -9 : … …. -4 -4 $T_n=-4n+3$	✓ first three terms ✓✓ answer (3)
3.4	$T_{15}=-4(15)+3$ $T_{15}=-57$	✓ substitution of $n=15$ ✓ answer (2)
3.5	$-145=-4n+3$ $4n=148$ $n=37$	✓ substitution of $T_n=-145$ ✓ answer (2)
		[10]

QUESTION 4

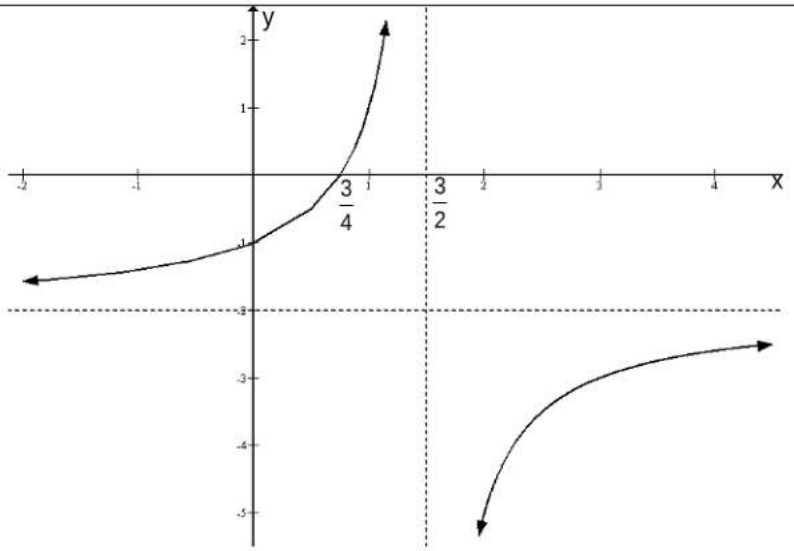
Given a quadratic sequence: 3 ; 17 ; 35 ; 57 ; . . .

4.1	next two terms : 83 and 113	✓ 83 ✓ 113 (2)
4.2	3 : 17 ; 35 57 .... 14 18 22 4 4 $2a = 4 \quad \therefore a = 2$ $3a + b = T_2 - T_1 \quad \therefore 3(2) + b = 14 \quad \therefore b = 8$ $a + b + c = T_1 \ \therefore 2 + 8 + c = 3 \ \therefore c = -7$ $\therefore T_n = 2n^2 + 8n - 7$ **OR** $2a = 4 \quad \therefore a = 2$ $c = 3 + 4 - 14 = -7$ $\therefore T_n = 2n^2 + bn - 7$ $3 = 2(1)^2 + b - 7$ $b = 8$ $\therefore T_n = 2n^2 + 8n - 7$	✓ $2a = 4$ ✓ $3(2) + b = 14$ ✓ $2 + 8 + c = 3$ ✓ general term (4) **OR** ✓ $2a = 4$ ✓ $c = 3 + 4 - 14$ ✓ $3 = 2(1)^2 + b - 7$ ✓ general term (4)
4.3	$2n^2 + 8n - 7 > 826$ $2n^2 + 8n - 833 > 0$ CVs: $n = \dfrac{-(8) \pm \sqrt{(8)^2 - 4(2)(-833)}}{2(2)}$ $n = 18{,}51 \quad \text{or} \quad n \neq -22{,}51 \ (n \in N)$   $n \geq 19$ $\therefore n = 20$ T_{20} is the second term greater than 826	✓ $T_n > 826$ ✓ critical values ✓ solving for n ✓ answer (4)

4.4	General term for the sequence of first differences: $T_n = 4n + 10$ $122 = 4n + 10$ $n = 28$ $\therefore$ T_{28} and T_{29} in quadratic seq. have a 1st difference of 122 **OR** $T_{n+1} - T_n = 122$ $2(n+1)^2 + 8(n+1) - 7 - (2n^2 + 8n - 7) = 122$ $2n^2 + 4n + 2 + 8n + 8 - 7 - 2n^2 - 8n + 7 = 122$ $4n = 112$ $n = 28$ $\therefore$ Between T_{28} and T_{29}	✓ $T_n = 4n + 10$ ✓ $122 = 4n + 10$ ✓ $n = 28$ ✓ T_{28} and T_{29} (4) **OR** ✓ $T_{n+1} - T_n = 122$ ✓ substitution ✓ $n = 28$ ✓ T_{28} and T_{29} (4)
4..5	$8p - 2 = -2(17)$ and $18q = -2(57)$ $8p - 2 = -34$ and $18q = -114$ $\therefore p = -4$ and $\therefore q = \frac{-19}{3}$ or $-6\frac{1}{3}$ or $-6,33$	✓ $p = -4$ ✓ $q = \frac{-19}{3}$ (2)
		[16]

QUESTION 5

Given: $h(x)=\dfrac{-3}{x-\dfrac{3}{2}}-2$

5.1.1	Equation of vertical asymptote: $x=\dfrac{3}{2}$ Equation of horizontal asymptote: $y=-2$	✓ $x=\dfrac{3}{2}$ ✓ $y=-2$ (2)
5.1.2	Coordinates of the y-intercept: $h(0)=\dfrac{-3}{2(0)-3}-2$ $y=-1$ $(0\ ;\ -1)$ Answer only: full marks	✓ $x=0$ ✓ $y=-1$ (2)
5.1.3	Coordinates of the x-intercept $0=\dfrac{-3}{2x-3}-2$ $2=\dfrac{-3}{2x-3}$ $2(2x-3)=-3$ $4x-6=-3$ $x=\dfrac{3}{4}$ $\left(\dfrac{3}{4}\ ;\ 0\right)$	✓ $h(x)=0$ ✓ simplification ✓ answer (3)
5.1.4		✓ both asymptotes ✓ both intercepts ✓ shape (3)

5.1.5	Eqn. of axis of symmetry of h that has a negative gradient: $y = -x + c$ $-2 = -\left(\frac{3}{2}\right) + c$ $c = -\frac{1}{2}$ $y = -x - \frac{1}{2}$ Answer only: full marks	✓ substitution of gradient of -1 and point $\left(\frac{3}{2}\ ;\ -2\right)$ ✓ answer (2)
5.1.6	Coordinates of points of $\cap$ of h and axis of symmetry: $\frac{-3}{x-\frac{3}{2}} - 2 = -x - \frac{1}{2}$ $\frac{-3}{x-\frac{3}{2}} = -x + \frac{3}{2}$ $\frac{-3}{x-\frac{3}{2}}\left(x-\frac{3}{2}\right) = \left(-x+\frac{3}{2}\right)\left(x-\frac{3}{2}\right)$ $-3 = -\left(x-\frac{3}{2}\right)^2$ $-3 = -x^2 + 3x - \frac{9}{4}$ $4x^2 - 12x - 3 = 0$ $x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(-3)}}{2(4)}$ $x = 3,23$ or $x = -0,23$ $y = -3,73$ or $y = -0,27$ $\therefore (-0,23\ ;\ -0,27)$ and $(3,23\ ;\ -3,73)$	✓ equating f and axis of symmetry ✓ standard form ✓ substitution into correct formula ✓ both x-values ✓ both y-values (5)
5.2.1	Equation of k: $q = 1$ $y = b^x + 1$ $\frac{3}{2} = b^1 + 1$ $b = \frac{1}{2}$ $k(x) = \left(\frac{1}{2}\right)^x + 1$ **OR** $k(x) = 2^{-x} + 1$	✓ value of q ✓ value of b ✓ equation of k (3)
5.2.2	Describe, in words, the transformation from k to $j(x) = 2^x$: Reflect k about the y –axis and translate 1 unit down (in any order)	✓ reflection about y –axis ✓ translation of 1 unit downwards (2)
		[22]

QUESTION 6

$f(x)=\frac{1}{2}(x-2)^2-8$ and $g(x)=x-6$

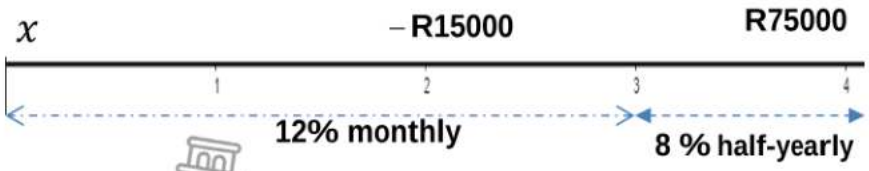
6.1	$f(x)=\frac{1}{2}(x-2)^2-8$ $0=\frac{1}{2}(x-2)^2-8$ $(x-2)^2=16$ $x-2=\pm 4$ $x_A=6 \quad \text{or} \quad x_B=-2$ $AB=6-(-2)$ $AB=8$ **OR** $f(x)=\frac{1}{2}(x-2)^2-8$ $0=\frac{1}{2}(x-2)^2-8$ $0=\frac{1}{2}x^2-2x-6$ $x^2-4x-12=0$ $(x-6)(x+2)=0$ $x_A=6 \quad \text{or} \quad x_B=-2$ $AB=6-(-2)$ $AB=8$	✓ $f(x)=0$ ✓ square root on both sides with $\pm$ ✓ both values of x ✓ length of AB (4) **OR** ✓ $f(x)=0$ ✓ standard form ✓ both values of x ✓ length of AB (4)
6.2	D(2 ; −8)	✓ answer (1)
6.3	Range of f: $y \geq -8$ **OR** $y \in [-8 ; \infty)$	✓ answer (1)
6.4	Average gradient between points A and C C(0 ; −6) $m=\frac{y_2-y_1}{x_2-x_1}$ $m=\frac{-6-0}{0-(-2)}$ $m=-3$	✓ coordinates of C ✓ correct substitution into gradient formula ✓ answer (3)

6.5	For 2 +ve roots of $\frac{1}{2}x^2-2x-6-k=0$: $-8<k<-6$ **OR** $x\in(-8\ ;\ -6)$	✓✓answer (2)
6.6.1	Values of x for which $f(x)\geq 0$: $x\leq -2$ or $x\geq 6$ **OR** $x\in(-\infty\ ;\ -2]\ \cup\ [6\ ;\ \infty)$	✓✓answer (2)
6.6.2	Values of x for which $\frac{g(x)}{f(x)}>0$: $x>-2\quad ;\ x\neq 6$ **OR** $x\in(-2\ ;\ 6)\ \cup\ (6\ ;\ \infty)$	✓✓answer (2)
6.7	Maximum value of h if $h(x)=\sqrt{2}^{2-f(x)}$: $2-f(x)$ is a max. when $f(x)$ is a min. min. of $f(x)=-8$ $\therefore 2-f(x)$ has a max. value of 10 max. of $h(x)=(\sqrt{2})^{10}$ $=2^5$ $=32$	✓ min. value of $f(x)=-8$ ✓ max. value of exponent = 10 ✓ answer (3)
6.8	Values of k for which $y=-3x+k$ will not be a tan. to f: $\frac{1}{2}(x-2)^2-8=-3x+k$ $\frac{1}{2}x^2-2x+2-8=-3x+k$ $x^2+2x-12-2k=0$ $a=1\quad b=2\quad c=-12-2k$ Line will be a tangent to f when $\Delta=0$: $(2)^2-4(1)(-12-2k)=0$ $4+48+8k=0$ $k=-\frac{13}{2}$ $\therefore k\neq -6\frac{1}{2}$	✓ equating both equations ✓ standard form in terms of k ✓ substitution into b^2-4ac ✓ value of k ✓ answer (5)
		[23]

QUESTION 7

7.1	$A = P(1-i)^n$ $A = R625\,000(1-0,1025)^6$ $A = R326\,653,08$	✓ substitution ✓ answer (2)
7.2	$1+i_{eff} = \left(1+\frac{i_{nom}}{m}\right)^m$ $i_{eff} = \left(1+\frac{0,08}{2}\right)^2 - 1$ $r_{eff} = \frac{51}{625} \times 100\%$ $r = 8,16\%$	✓ formula ✓ substitution ✓ answer (3)
7.3	$A = P(1+i)^n$ $1\,671\,650,74 = P\left(1+\frac{0,14}{4}\right)^{4\times4,5}$ $P = \frac{1\,671\,650,74}{\left(\frac{207}{200}\right)^{18}}$ $P = R899\,951,80$	✓ $n = 4\times4,5 = 18$ ✓ substitution into correct formula ✓ answer (3)
7.4.1	$75\,000 = P_3\left(1+\frac{0,08}{2}\right)^{1\times2}$ $P_3 = R69\,341,72$	✓ $n = 2$ ✓ $i = \frac{0,08}{2}$ or $\frac{8\%}{2}$ ✓ answer (3)

7.4.2



$$75\,000 = \left[x\left(1+\frac{0,12}{12}\right)^{36} - 15\,000\left(1+\frac{0,12}{12}\right)^{12}\right]\left(1+\frac{0,08}{2}\right)^{2}$$

✓ $x\left(1+\frac{0,12}{12}\right)^{36}$

✓ $-15\,000\left(1+\frac{0,12}{12}\right)^{12}$

✓ $\frac{75\,000}{\left(1+\frac{0,08}{2}\right)^{2}}$

$$x = \frac{\frac{75\,000}{\left(1+\frac{0,08}{2}\right)^{2}} + 15\,000\left(1+\frac{0,12}{12}\right)^{12}}{\left(1+\frac{0,12}{12}\right)^{36}}$$

x = R60 278,15

✓ answer (4)

OR

OR

Amount in account after R1 500 was withdrawn $= \frac{69\,341,72}{\left(1+\frac{0,12}{12}\right)^{12}}$

= R61 537,25569

Amount after withdrawal:

✓ $\frac{69\,341,72}{\left(1+\frac{0,12}{12}\right)^{12}}$

Amt. in acc. before R1500 was withdrawn = 61537,25569 + 15000

i.e. value after 2 years = R76 537,25569

Amount before withdrawal

✓ R61537,2521 + R15000

$x\left(1+\frac{0,12}{12}\right)^{24} = 76\,537,25569$

✓ equation

$x = \frac{76\,537,25569}{\left(1+\frac{0,12}{12}\right)^{24}}$

x = R60 278,15

✓ answer (4)

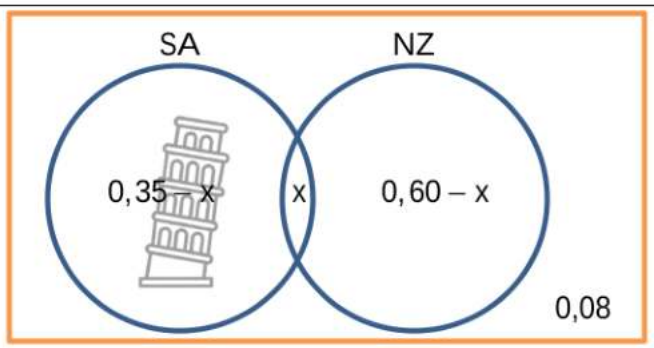
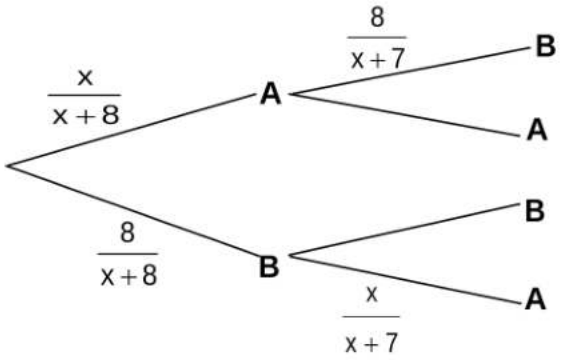
[15]

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QUESTION 8

8.1.1	$P(A \text{ or } B) = P(A) + P(B)$ $= 0,35 + 0,40$ $= 0,75$	✓ $0,35 + 0,40$ ✓ answer (2)
8.1.2	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $= 0,35 + 0,40 - 0,35 \times 0,40$ $= 0,61$	✓ $P(A \text{ and } B) = 0,35 \times 0,40 = 0,14$ ✓ correct substitution into rule ✓ answer (3)
8.2.1	$k = 18$	✓ answer (1)
8.2.2	$P(\text{Boy who does not listen to Pop music}) = \frac{32 + 23}{150}$ $= \frac{11}{30} \approx 0,37$	✓✓answer (2)
8.2.3	$P(\text{Girl and listens to Pop muic}) = \frac{18}{150}$ $= 0,12$ $P(\text{Girl}) \times P(\text{listens to Pop music}) = \frac{80}{150} \times \frac{33}{150}$ $= 0,12$ $P(\text{Girl and listens to Pop}) = P(\text{Girl}) \times P(\text{listens to Pop})$ $\therefore$ The events are independent.	✓ $\frac{18}{150}$ or $0,12$ ✓ $\frac{80}{150} \times \frac{33}{150}$ ✓ $0,12$ ✓ conclusion (4)
		[12]

QUESTION 9

9.1	 $0,35 - x + x + 0,60 - x + 0,08 = 1$ $x = 0,03$ P(only NZ will reach the finals) $= 0,6 - 0,03$ $= 0,57$ **OR** P(NZ or SA) = P(NZ) + P(SA) – P(NZ and SA) $1 - 0,08 = 0,35 + 0,6 - P(NZ \text{ and } SA)$ P(NZ and SA) = 0,03 P(only NZ will reach the finals) $= 0,6 - 0,03$ $= 0,57$	✓ $0,35 - x$ ✓ $0,60 - x$ ✓ sum of probabilities = 1 ✓ P(only NZ) $= 0,57$ (4) **OR** ✓ P(NZ or SA) = 0,92 ✓ correct substitution into rule ✓ P(NZ and SA) = 0,03 ✓ P(only NZ) = 0,57 (4)
9.2	 $\left(\frac{x}{x+8}\right)\left(\frac{8}{x+7}\right)+\left(\frac{8}{x+8}\right)\left(\frac{x}{x+7}\right)=\frac{48}{95}$ $\frac{8x+8x}{(x+8)(x+7)}=\frac{48}{95}$ $\frac{16x}{x^2+15x+56}=\frac{48}{95}$ $48x^2+720x+2688=1520x$ $3x^2-50x+168=0$ $x=\frac{-(-50)\pm\sqrt{(-50)^2-4(3)(168)}}{2(3)}$ $x=12 \quad \text{or} \quad x\neq\frac{14}{3}$ There are 12 apples in the basket	✓ $P(A).P(B)=\left(\frac{x}{x+8}\right)\left(\frac{8}{x+7}\right)$ ✓ $P(B).P(A)=\left(\frac{8}{x+8}\right)\left(\frac{x}{x+7}\right)$ ✓ $P(A).P(B)+P(B).P(A)=\frac{48}{95}$ ✓ standard form ✓ answer
		[09]

TOTAL: 150

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