GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION

2023

MARKING GUIDELINES

MATHEMATICS (PAPER 1) (10611)

18 pages

INSTRUCTIONS AND INFORMATION:

A – Accuracy

CA – Continued Accuracy

S – Statement

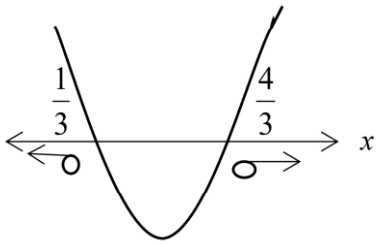
R – Reason

S and R – Statement and Reason

NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed OUT an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- It is UNACCEPTABLE to assume values/answers in order to solve a question.

QUESTION 1

1.1	1.1.1	$(2x+1)^2-4=0$ $4x^2+4x+1-4=0$ $4x^2+4x-3=0$ $(2x+3)(2x-1)=0$ $x=-\frac{3}{2}\cdots or\cdots x=\frac{1}{2}$ **OR** $(2x+1)^2-4=0$ $2x+1=\pm\sqrt{4}$ $2x+1=\pm 2$ $2x=\pm 2-1$ $2x=2-1\cdots or\cdots 2x=-2-1$ $2x=1\cdots or\cdots 2x=-3$ $x=\frac{1}{2}\cdots or\cdots x=-\frac{3}{2}$ **NOTE: In option 2, the simplification mark must only be awarded if the candidate states ± 2 or demonstrates it later in the solution.**	✓ standard form ✓ factors ✓ answers ✓ find root ✓ simplification ✓ answers	(3)
	1.1.2	$4x^2-11=-12x$ $4x^2+12x-11=0$ $x=\frac{-12\pm\sqrt{(12)^2-4(4)(-11)}}{2(4)}$ $x=-3,74$ or $x=0,74$ **NOTE: Penalise 1 mark for rounding in this question only. Candidate must show the substitution to obtain full marks.**	✓ correct substitution ✓✓ answers	(3)
	1.1.3	$15x-4<9x^2$ $\therefore -9x^2+15x-4<0$ $\therefore 9x^2-15x+4>0$ $(3x-1)(3x-4)>0$ $\therefore x<\frac{1}{3}\cdots or\cdots x>\frac{4}{3}$  **NOTE: Penalise 1 mark for use of AND instead of OR**	✓ standard form ✓ factors ✓✓ answers	(4)

	1.1.4	$\sqrt{2x-2}-\sqrt{7-2x}=1$ $(\sqrt{2x-2})^2=(\sqrt{7-2x}+1)^2$ $2x-2=7-2x+2\sqrt{7-2x}+1$ $4x-10=2\sqrt{7-2x}$ $(2x-5)^2=(\sqrt{7-2x})^2$ $4x^2-20x+25=7-2x$ $4x^2-18x+18=0$ $\therefore 2x^2-9x+9=0$ $(x-3)(2x-3)=0$ $\therefore x=3\cdots or\cdots x=\frac{3}{2}$ $reject\cdots x\neq\frac{3}{2}$	✓ isolate surd and square both sides ✓ simplification ✓ standard form ✓ factors ✓ $x=3$ and rejection	(5)
1.2	$a^2b^2-2ab-8=0$ and $\log_2(a+5)=3$ $\log_2(a+5)=3$ $2^3=a+5$ $8=a+5$ $a=3$ $\therefore a^2b^2-2ab-8=0$ $(3)^2b^2-2(3)b-8=0$ $9b^2-6b-8=0$ $(3b-4)(3b+2)=0$ $b=\frac{4}{3}$ or $b=-\frac{2}{3}$ **OR** $a^2b^2-2ab-8=0$ and $\log_2(a+5)=3$ $\log_2(a+5)=3$ $2^3=a+5$ $8=a+5$ $a=3$ $a^2b^2-2ab-8=0$ $(ab-4)(ab+2)=0$ $ab=4$ or $ab=-2$ $\therefore 3b=4$ $3b=-2$ $b=\frac{4}{3}$ or $b=-\frac{2}{3}$	✓ exponential form ✓ value of a ✓ substitution ✓ factors ✓ both answers for b **OR** ✓ exponential form ✓ value of a ✓ factors ✓ substitution ✓ both answers for b	(5)

1.3	$p=\dfrac{\sqrt{x+2}}{\sqrt{16-x^2}}$ $\therefore x+2\geq 0$ $\therefore x\geq -2\cdots\cdots\cdots(1)$ $\therefore 16-x^2>0$ $(4+x)(4-x)>0$ $\therefore -4<x<4\cdots\cdots\cdots(2)$ From (1) and (2) $-2\leq x<4$	✓ equation (1) ✓ factors ✓ equation (2) ✓ answer	(4)
			[24]

QUESTION 2

2.1	2.1.1	$1-p;\ 2p-3;\ p+5$ $d=T_2-T_1=T_3-T_2$ $\therefore (2p-3)-(1-p)=(p+5)-(2p-3)$ $2p-3-1+p=p+5-2p+3$ $3p-4=-p+8$ $4p=12$ $\therefore p=3$		✓ substitution ✓ answer	(2)
	2.1.2	(a)	$T_1=1-p$ $T_1=1-3$ $T_1=-2$	✓ answer	(1)
		(b)	$T_2=2p-3$ $T_2=2(3)-3$ $\therefore T_2=3$ $\therefore d=T_2-T_1$ $d=3-(-2)$ $d=5$	✓ answer	(1)
	2.1.3	$1-p;\cdots 2p-3;\cdots p+5;...$ $=1-3;\cdots 2(3)-3;\cdots 3+5;...$ $=-2;\cdots\cdots 3;\cdots\cdots 8;\cdots 13;\cdots 18;\cdots$ All the terms except T_1 end in either 3 or 8 while perfect squares end on 1; 4; 9; 6; 5; 0.		✓ correct terms ✓ explanation	(2)

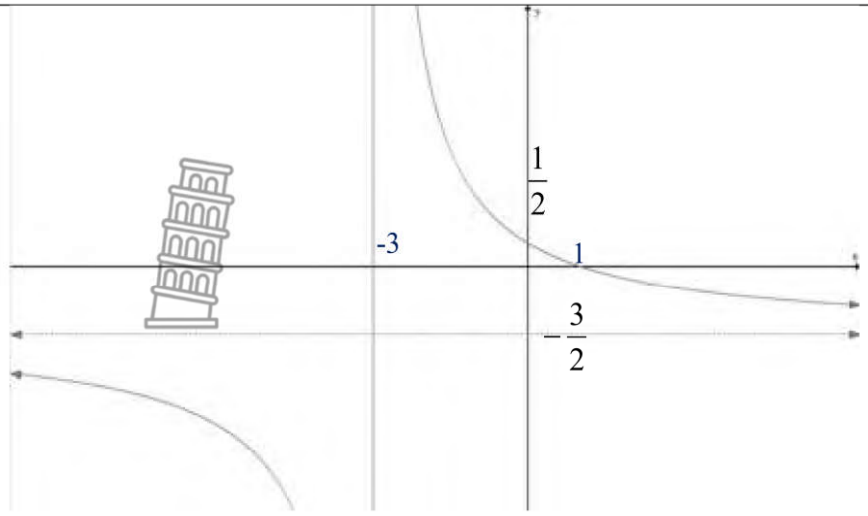
2.2	2.2.1	$-3;\ -2;\ -3;\ -6;\ -11;\ldots$ First differences: $+1;\ -1;\ -3;\ -5$ $T_n = a+(n-1)d$ $T_n = 1+(n-1)(-2)$ $T_n = 1-2n+2$ $T_n = -2n+3$	✓ first differences ✓ substitution ✓ answer	(3)
	2.2.2	$T_n = -2n+3$ $T_{35} = -2(35)+3$ $\therefore T_{35} = -67$	✓ answer	(1)
	2.2.3	-3 -2 -3 -6 -11 1 -1 -3 -5 (1st differences) -2 -2 -2 (2nd differences) $T_n = an^2+bn+c$ $but\cdots 2a=-2$ $\therefore a=-1$ $\therefore T_n = -n^2+bn+c$ $T_1 = -(1)^2+b(1)+c$ $-3 = -1+b+c$ $\therefore -2 = b+c\cdots\cdots(1)$ $T_2 = -(2)^2+b(2)+c$ $-2 = -4+2b+c$ $\therefore 2 = 2b+c\cdots\cdots(2)$ $(2-1)\cdots\therefore b=4$ $sub(1)\cdots -2 = 4+c$ $\therefore c=-6$ $\therefore T_n = -n^2+4n-6$	✓ second differences ✓ $a=-1$ ✓ $b=4$ ✓ $c=-6$	(4)
	2.2.4	$T_n = -n^2+4n-6$ $T_n = -[n^2-4n+2^2-4+6]$ $T_n = -[(n-2)^2+2]$ $\therefore T_n = -(n-2)^2-2$ $\therefore T_n(\max) = -2$ $\therefore$ NO positive terms.	✓ method ✓ $T_n(\max)$	(2)
				[16]

QUESTION 3

3.1	$S_n = 4n^2 + 1$ $S_6 = 4(6)^2 + 1$ $\therefore S_6 = 145$ $S_5 = 4(5)^2 + 1$ $\therefore S_5 = 101$ $\therefore T_6 = S_6 - S_5$ $\therefore T_6 = 145 - 101$ $\therefore T_6 = 44$	✓ value of S_6 ✓ value of S_5 ✓ answer	(3)
3.2	$(4x-3) + (4x-3)^2 + (4x-3)^3$ $\therefore r = 4x - 3$ $\therefore -1 < r < 1; \cdots\cdots r \neq 0$ $-1 < 4x - 3 < 1; \cdots\cdots\cdots 4x - 3 \neq 0$ $\therefore 2 < 4x < 4 \cdots\cdots\cdots x \neq \frac{3}{4}$ $\therefore \frac{1}{2} < x < 1; \cdots\cdots\cdots x \neq \frac{3}{4}$ **NOTE: No penalty if candidate does not state: $x \neq \frac{3}{4}$**	✓ $r = 4x - 3$ ✓ $-1 < r < 1$ ✓ answer	(3)
3.3	$\sum_{k=3}^{5} (-1)^k . \frac{2}{k}$ $= (-1)^3 . \frac{2}{3} + (-1)^4 . \frac{2}{4} + (-1)^5 . \frac{2}{5}$ $= -\frac{2}{3} + \frac{1}{2} - \frac{2}{5}$ $= -\frac{17}{30}$ **NOTE: Answer only, full marks.**	✓ expansion ✓ answer	(2)
			[8]

QUESTION 4

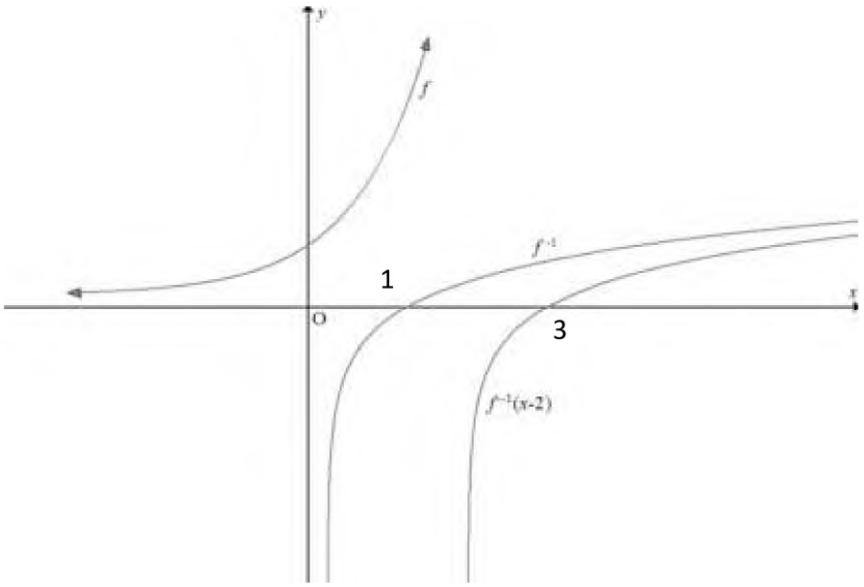
4.1	$x \in \Re;\quad x \neq -3$ **NOTE: Both conditions must be stated.**		✓ answer	(1)
4.2	$y \in \Re;\quad y \neq 2$ **NOTE: Both conditions must be stated.**		✓ answer	(1)
4.3	4.3.1	6 units right. **NOTE: Accept 6 units, but not 6 units left.**	✓ answer	(1)
	4.3.2	3,5 units upwards. **NOTE: Accept 3,5 units, but not 3,5 units downwards.**	✓ answer	(1)
4.4	$x = -3$ $y = -\frac{3}{2}$ **NOTE: Do not accept any equating in terms of *p* and *q*.**		✓ answer ✓ answer	(2)
4.5	$0 = \frac{6}{x+3} - 1,5$ $\frac{3}{2} = \frac{6}{x+3}$ $3(x+3) = 6(2)$ $3x + 9 = 12$ $3x = 3$ $x = 1$ $(1;0)$ **NOTE: The answer does NOT need to be given in coordinate form.**		✓ answer	(1)

4.6		✓ asymptotes ✓ intercepts ✓ shape	(3)
4.7	$y=-x+k$ $\therefore -\frac{3}{2}=-(-3)+k$ $-\frac{3}{2}=3+k$ $\therefore k=-\frac{9}{2}$	✓ substitute point ✓ answer	(2)
4.8	$x>-3$	✓ answer	(1)
4.9	$y=\frac{-6}{x+3}+\frac{3}{2}$ **OR** $y=\frac{6}{-x-3}+\frac{3}{2}$ **OR** $y=-\frac{6}{x+3}+\frac{3}{2}$	✓ answer	(1)
			[14]

QUESTION 5

5.1	$\mathrm{E}\left(\frac{7}{2};\frac{81}{4}\right)$	✓ answer	(1)
5.2	$f(x)=-\left(x-\frac{7}{2}\right)^2+\frac{81}{4}$ $f(1)=-\left(1-\frac{7}{2}\right)^2+\frac{81}{4}=14$ $f(5)=-\left(5-\frac{7}{2}\right)^2+\frac{81}{4}=18$ $\therefore AG=\frac{f(5)-f(1)}{5-1}$ $=\frac{18-14}{4}$ $\therefore AG=1$	✓ value of $f(1)$ ✓ value of $f(5)$ ✓ substitution ✓ answer	(4)
5.3	$f(x)=-(x-\frac{7}{2})^2+\frac{81}{4}$ $\therefore f(x)=-x^2+7x+8$ $\because f(x)=g(x)$ $\therefore -x^2+7x+8=-3x+24$ $-x^2+10x-16=0$ $\therefore x^2-10x+16=0$ $(x-2)(x-8)=0$ $\therefore x=2\cdots or\cdots x=8$ $\therefore x_D=2$	✓ equating ✓ x-values ✓ answers/selection	(3)
5.4	$ST=f(x)-g(x)$ $ST=-x^2+7x+8-(-3x+24)$ $ST=-x^2+7x+8+3x-24$ $ST=-x^2+10x-16$	✓ method ✓ answer	(2)
5.5	For max: $\frac{d(ST)}{dx}=0$ $\frac{d(ST)}{dx}=-2x+10=0$ $2x=10$ $\therefore x=5$ max: $ST=-(5)^2+10(5)-16$ $\therefore ST=9$	✓ derivative = 0 ✓ value for x ✓ answer	(3)
			[13]

QUESTION 6

6.1	$y = 3^x$ Inverse $x = 3^y$ $\therefore y = \log_3 x$	✓ answer	(1)
6.2	$y = f^{-1}(x) = \log_3 x$ *x*-intercept: $\log_3 x = 0$ $\therefore x = 3^0$ $\therefore x = 1$ *y*-intercept: None $y = f^{-1}(x-2) = \log_3(x-2)$ *x*-intercept: $\log_3(x-2) = 0$ $\therefore x - 2 = 3^0$ $\therefore x = 3$ *y*-intercept: None  **NOTE: If the candidate does not draw the graph but calculates both *x*-intercepts, award 2 marks.**	✓ *x*-intercept of $f^{-1}(x) = \log_3 x$ ✓ shape $f^{-1}(x) = \log_3 x$ ✓ *x*-intercept of $f^{-1}(x-2) = \log_3(x-2)$ ✓ shape $f^{-1}(x-2) = \log_3(x-2)$	(4)
6.3	$\log_3(x-2) < 1$ $\therefore 2 < x < 5$	✓ critical values ✓ correct notation	(2)
			[7]

QUESTION 7

7.1	$A = P(1+i)^n$ $6\,000\,000 = 5\,700\,000(1+i)^7$ $\frac{6\,000\,000}{5\,700\,000} = (1+i)^7$ $\sqrt[7]{1{,}0526\ldots} = 1+i$ $\therefore i = 1{,}00735 - 1$ $\therefore i = 0{,}00735$ rate: 0,74%	✓ substitution into correct formula ✓ value of i ✓ answer	(3)
7.2	7.2.1 $P_v = \frac{x[1-(1+i)^{-n}]}{i}$ $4\,000\,000 = \frac{30\,000\left[1-\left(1+\frac{0,06}{12}\right)^{-n}\right]}{\frac{0,06}{12}}$ $\frac{400\,000\times\left(\frac{0,06}{12}\right)}{30\,000} = 1-\left(1+\frac{0,06}{12}\right)^{-n}$ $\frac{1}{3} = \left(1+\frac{0,06}{12}\right)^{-n}$ $\log_{\left(1+\frac{0,06}{12}\right)}\frac{1}{3} = -n$ $n = 220,27$ $\therefore$ she will make 220 withdrawals.	✓ substitution into correct formula ✓ $i = \frac{0,06}{12}$ ✓ simplification ✓ correct use of logs ✓ answer of 220	(5)

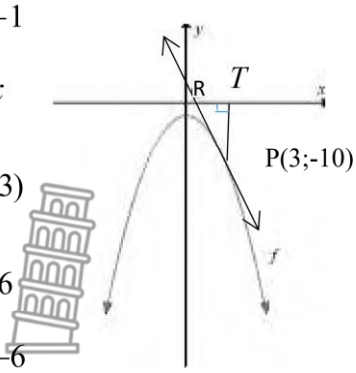
	7.2.2	$4\,000\,000 = \dfrac{20\,000\left[1-\left(1+\dfrac{0,06}{12}\right)^{-n}\right]}{\dfrac{0,06}{12}}$ $0 = \left(1+\dfrac{0,06}{12}\right)^{-n}$ She can make any number (an infinite number) of withdrawals. Her interest earned equals her withdrawal amount. She will only be drawing the interest amount. **OR** $A = P(1+i)^n$ $A = 4\,000\,000\left(1+\dfrac{0.06}{12}\right)^1$ $A = \text{R}4\,020\,000$ She can make any number (an infinite number) of withdrawals. Her interest earned equals her withdrawal amount. She will only be drawing the interest amount.	✓ valid method ✓ simplification ✓ explanation **OR** ✓ valid method ✓ answer ✓ explanation	(3)
7.3	$A = P(1+i)^n$ $A = 1\,000(1+\dfrac{0,15}{12})^{18}$ $A = \text{R}1\,250,58$ $F_v = \dfrac{x[(1+i)^n - 1]}{i}$ $F_v = \dfrac{700[(1+\dfrac{0,15}{12})^{18} - 1]}{\dfrac{0,15}{12}}$ $F_v = \text{R}14\,032,33$ $\text{Amount} = \text{R}1\,250,58 + \text{R}14\,032,33$ $\therefore \text{Amount} = \text{R}15\,282,91$	✓ value of A ✓ substitution in correct formula ✓ value for F_v ✓ answer	(4)
				[15]

QUESTION 8

8.1	$f(x)=3x^2-6$ $f(x+h)=3(x+h)^2-6$ $f(x+h)=3(x^2+2xh+h^2)-6$ $f(x+h)=3x^2+6xh+3h^2-6$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{3x^2+6xh+3h^2-6-3x^2+6}{h}$ $f'(x)=\lim_{h\to 0}\frac{6xh+3h^2}{h}$ $f'(x)=\lim_{h\to 0}\frac{h(6x+3h)}{h}$ $f'(x)=\lim_{h\to 0}6x+3h$ $f'(x)=6x$ **NOTE: Penalise 1 mark for notation error in this question only.**	✓ $f(x+h)$ ✓ substitution ✓ factorisation ✓ answer	(4)
8.2	$f(x)=(2\sqrt{x}-\frac{1}{x})^2$ $f(x)=4x-4.\frac{1}{\sqrt{x}}+\frac{1}{x^2}$ $f(x)=4x-4x^{-\frac{1}{2}}+x^{-2}$ $\frac{d_y}{d_x}=4+2x^{-\frac{3}{2}}-2x^{-3}$ **OR** $\frac{d_y}{d_x}=4+\frac{2}{\sqrt{x^3}}-\frac{2}{x^3}$	✓ simplification ✓ $f(x)=4x-4x^{-\frac{1}{2}}+x^{-2}$ ✓ 4 ✓ $2x^{-\frac{3}{2}}$ or $\frac{2}{\sqrt{x^3}}$ ✓ $-2x^{-3}$ or $\frac{2}{x^3}$	(5)
8.3	$f(x)=3x^3-3x^2+6x-2$ $f'(x)=9x^2-6x+6$ $f''(x)=18x-6$ for concave up $18x-6>0$ $\therefore x>\frac{1}{3}$	✓ $f'(x)$ ✓ $f''(x)$ ✓ correct condition of concavity ✓ answer	(4)
			[13]

QUESTION 9

9.1	9.1.1	$f'(x) = -6x^2 - 6x + 12$ y-intercept $x = 0$ $\therefore f'(0) = 12$ $\therefore$ A(0 ; 12) **NOTE: Does not have to be in coordinate form.**	✓ answer	(1)
	9.1.2	x-intercepts $y = f'(x) = 0$ $-6x^2 - 6x + 12 = 0$ $\therefore x^2 + x - 2 = 0$ $(x+2)(x-1) = 0$ $\therefore x = -2 \cdots or \cdots x = 1$ $\therefore$ B(−2 ; 0) ; C(1 ; 0) **NOTE: Must be in coordinate form.**	✓ factors ✓ coordinates of B ✓ coordinates of C	(3)
	9.1.3	Turning points	✓ answer	(1)
	9.1.4	$f(x)$ increases where: $m = f'(x) > 0$ $\therefore -2 < x < 1$ **OR** $m = f'(x) > 0$ $\therefore x \in (-2 ; 1)$	✓ $m = f'(x) > 0$ ✓✓ answers **OR** ✓ $m = f'(x) > 0$ ✓✓ answers	(3)
	9.1.5	Point of inflection: $f''(x) = 0$ $\therefore -12x - 6 = 0$ $-12x = 6$ $\therefore x = -\frac{1}{2}$ $f(x) = -2x^3 - 3x^2 + 12x + 20$ $\therefore f(-\frac{1}{2}) = -2(-\frac{1}{2})^3 - 3(-\frac{1}{2})^2 + 12(-\frac{1}{2}) + 20$ $\therefore f(-\frac{1}{2}) = 13{,}5$	✓ $f''(x) = 0$ ✓ value for x ✓ substitution ✓ value for $f(-\frac{1}{2})$	(4)

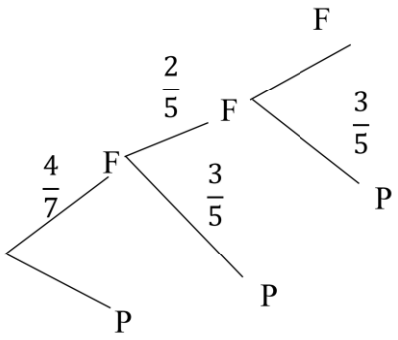
9.2	$y=-x^2-1$ $\frac{d_y}{d_x}=-2x$ $\frac{d_y}{d_x}=-2(3)$ $\therefore \frac{d_y}{d_x}=-6$ $\therefore m_{PR}=-6$ $\because \frac{PT}{RT}=-6$ $\therefore \frac{-10}{RT}=-6$ $-6RT=-10$ $\therefore RT=\frac{5}{3}$		✓ derivative ✓ $\frac{d_y}{d_x}=-6$ ✓ $m_{PR}=-6$ ✓ substitution ✓ answer
			(5)
			[17]

QUESTION 10

10.	$W(x)=-\frac{x^3}{150}+3x^2-250x-2700$ $\therefore$ max : $W'(x)=-\frac{3x^2}{150}+6x-250=0$ $-x^2+300x-12500=0$ $\therefore x^2-300x+12500=0$ $(x-250)(x-50)=0$ $\therefore x=250\cdots or\cdots x=50$ $W''(x)=-\frac{6x}{150}+6$ $W''(x)=-\frac{x}{25}+6$ $\therefore W''(50)=-\frac{50}{25}+6>0$ $\therefore W''(250)=-\frac{250}{25}+6<0$ $\therefore$ Maximum profit: 250 bicycles weekly.		✓ $W'(x)$ ✓ standard form ✓ factors ✓ values of x ✓ $W''(x)$ ✓ $W''(50)>0$ ✓ $W''(250)<0$ ✓ conclusion
			[8]

QUESTION 11

NOTE: Candidates can present solutions either in decimal or fraction form. Do not penalise for rounding-off.

11.1	11.1.1	For independent events: $P(A) \times P(B) = P(A \text{ en } B)$ $\therefore (x + 0,1) \times (0,4) = (0,1)$ $\therefore (x + 0,1) = 0,25$ $\therefore x = 0,15$ $x + 0,1 + 0,3 + y = 1$ $0,15 + 0,1 + 0,3 + y = 1$ $\therefore y = 0,45$	✓ correct substitution ✓ value for x ✓ method ✓ value for y	(4)
	11.1.2	$0,15 + 0,3$ $= 0,45$ **NOTE: Answer only, full marks.**	✓ answer	(1)
11.2	 F (unsuccessful)/P (pass/succeed)			
	11.2.1	$\frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$ **NOTE: Answer only, full marks.**	✓ method ✓ answer	(2)
	11.2.2	$\frac{4}{7} \times \frac{2}{5} \times \frac{3}{5} = \frac{24}{175}$ **NOTE: Answer only, full marks.**	✓ method ✓ answer	(2)
				[9]

QUESTION 12

12.1	12.1.1	1 **NOTE: Accept 2.**	✓ answer	(1)
	12.1.2	$7!-(2!\times 6!) = 3\,600$ **NOTE: Accept answer as 3 600 or 7!–(2!×6!)** **No CA if method is meaningless.**	✓ 7! ✓ –(2!×6!) ✓ answer	(3)
12.2	P(win) $= 0,7 \times 0,9 + 0,3 \times 0,45$ $= 0,765$ $= 76,5\%$ **NOTE: No CA if method is meaningless.**		method answer	(2)
				[6]
				TOTAL: 150