GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JOHANNESBURG WEST DISTRICT

TERM 1
CONTROLLED TEST
01 MARCH 2023

GRADE 12

MATHEMATICS

Stanmorephysics

MARKS: 50 **DURATION: 1 HOUR**

This question paper consists of 6 pages including the formula sheet.

INSTRUCTIONS AND INFORMATION

1. This question paper consists of 6 questions.
2. Answer ALL the questions in your answer book.
3. Use the appropriate and correct numbering system as it is used on this paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
7. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of this paper.
10. It is in your own interest to write legibly and to present your work neatly.

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QUESTION 1

1.1 Solve for x.

1.1.1 $x(2x-3)=0$ (2)

1.1.2 $0=3x^2-5x-11$ (3)

1.1.3 $x(3x+4)-2(3x+4)\leq 0$ (3)

1.2 The roots of a quadratic equation, in terms of p, are given as:

$$x=\frac{4\pm\sqrt{8-p^3}}{p}$$

Determine the value(s) of p for the roots to be real. (3)

[11]

QUESTION 2

The first four (4) terms of a quadratic pattern are: $11\ ;\ 20\ ;\ 33\ ;\ 50\ ;\ ...$

2.1 Determine the general term of this pattern in the form $T_n=an^2+bn+c$. (4)

2.2 Prove that the sum of the first n first-differences of this quadratic pattern can be given by $S_n=2n^2+7n$. (2)

[6]

QUESTION 3

A convergent geometric series is given by: $\frac{5(x+1)}{3}+\frac{5(x+1)^2}{9}+\frac{5(x+1)^3}{27}+...$

3.1 Calculate the values of x. (3)

3.2 If $x=1$, calculate the sum to infinity, S_∞. (2)

[5]

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QUESTION 4

$(1-x)$; $(x+2)$ and $(2x-5)$ are the first three (3) terms of an arithmetic sequence.

4.1 Determine the value of x. (3)

4.2 If the first three (3) terms of this pattern are: 9 ; -6 ; -21 ; ... , calculate the numerical value of the sum of the first 100 terms, S_{100}. (2)

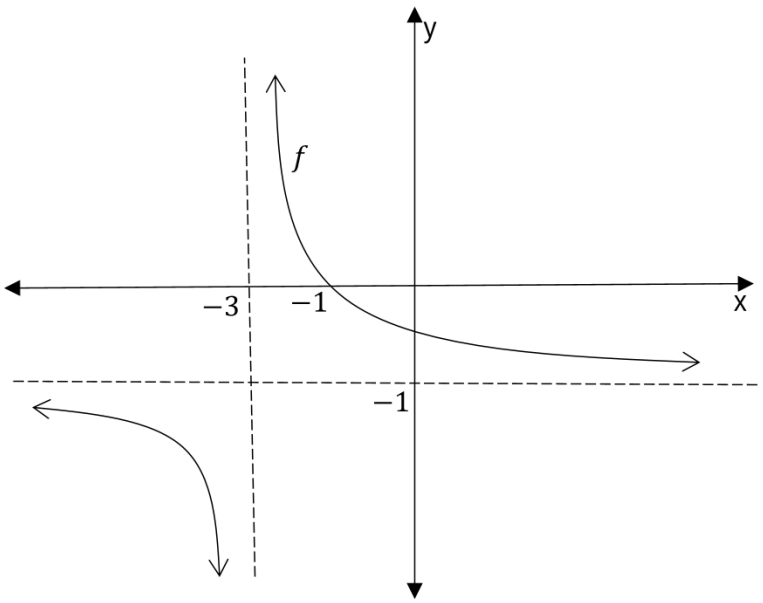
4.3 Hence, or otherwise, calculate m if : (4)

$$\sum_{n=0}^{m-1}(24-15n) = S_{100} + 73\,320$$

[9]

QUESTION 5

The graph of $f(x) = \frac{2}{x+p} + q$ is sketched below:



5.1 Write down the values of p and q (2)

5.2 The straight line $g(x) = -x + k$ is one of the axes of symmetry of the graph of f. Determine the value of k. (2)

5.3 If $h(x) = -2[g(x)]$, determine the equation of the inverse of h, h^{-1}, in the form $h^{-1}(x) = mx + c$. (3)

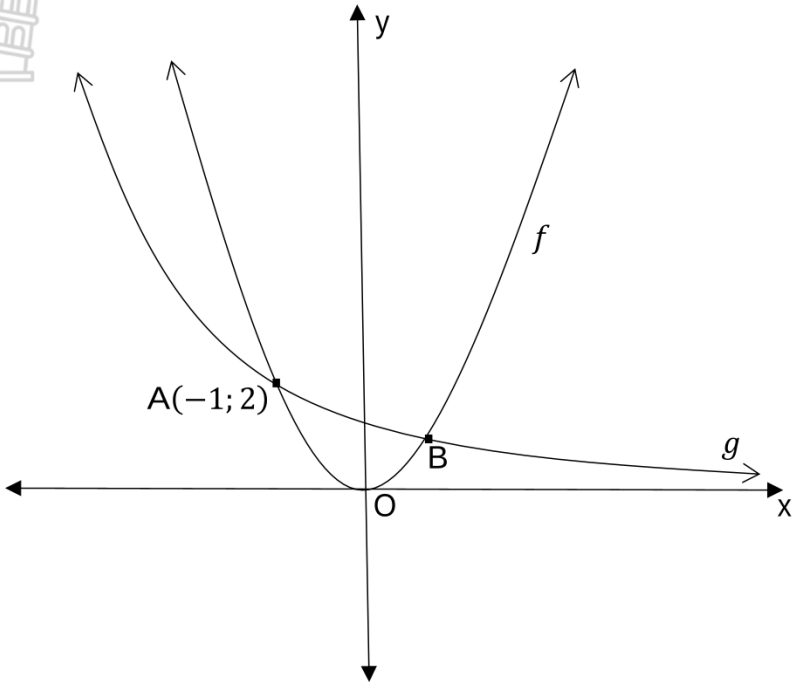
5.4 Draw a neat sketch of the graphs of h and h^{-1} on the same set of axes. Clearly show all intercepts with axes, point of intersection and the axis of symmetry. (4)

[11]

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QUESTION 6

The graphs of $f(x) = ax^2$ and $g(x) = b^x$ are sketched on the same set of axes. Points $A(-1; 2)$ and B are points of intersection of f and g. The graph of f has the turning point at the origin:



6.1 Calculate the values of a and b. (2)

6.2 The inverse of f is NOT a function. Write down at least one condition which can be used to restrict the domain of f such that its inverse will be a function. (1)

6.3 For which value(s) of x, where $x \in (-\infty; 0]$, will $g(x) \leq f(x)$? (2)

6.4 If $h(x) = g(x + 3)$, write down the coordinates of …

6.4.1 A', the new coordinates of A on the graph of h. (1)

6.4.2 A'', the new coordinates of A on the graph of h^{-1}, the inverse of h. (2)

[8]

TOTAL = 50 MARKS

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \;; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum f\dot{x}}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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