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GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JOHANNESBURG WEST DISTRICT

MARKING MEMORANDUM
CONTROLLED TEST 1
01 MARCH 2023

GRADE 12

MATHEMATICS

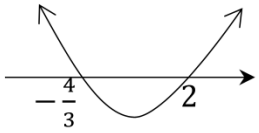
MARKS: 50

This Marking Guidelines consists of 6 pages including this cover page.

IMPORTANT NOTES AND INFORMATION

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Accept any other Mathematically valid attempt which yields a correct answer and credit full marks.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

QUESTION 1

Q#	Suggested Solutions	Descriptors
1.1.1	$x(2x-3)=0$ $x=0$ or $x=\frac{3}{2}$	✓ $x=0$ ✓ $x=\frac{3}{2}$ (2)
1.1.2	$0=3x^2-5x-11$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(-11)}}{2(3)}$ $x=-1{,}25$ or $x=2{,}92$	✓ correct substitution ✓ $x=-1{,}25$ ✓ $x=2{,}92$ (3)
1.1.3	$x(3x+4)-2(3x+4)\leq 0$ $(x-2)(3x+4)\leq 0$ **Critical values**: $x=2$; $x=-\frac{4}{3}$ $-\frac{4}{3}\leq x\leq 2$ 	✓ factors/ method ✓ critical values ✓ solution (3)
1.2	For real roots: $8-p^3\geq 0$; $p\neq 0$ $-p^3\geq -8$ $p^3\leq 8$ $p\leq 2$	✓ $8-p^3\geq 0$ ✓ $p\neq 0$ ✓ $p\leq 2$ (3)
	Subtotal	**[11]**

QUESTION 2

Q#	Suggested Solutions	Descriptors
2.1	$\quad 11 \;;\; 20 \;;\; 33 \;;\; 50$ 1st differences: $\quad 9 \;;\; 13 \;;\; 17$ 2nd differences: $\quad 4 \;;\; 4$ $\therefore\ 2a=4 \qquad 3a+b=9 \qquad a+b+c=11$ $a=2 \qquad 3(2)+b=9 \qquad 2+3+c=11$ $b=3 \qquad c=6$ $\therefore\ T_n=2n^2+3n+6$	✓ $a=2$ ✓ $b=3$ ✓ $c=6$ ✓ $T_n=2n^2+3n+6$ (4)
2.2	1st differences: $\quad 9 \;;\; 13 \;;\; 17$ $a=9\,;\,d=4$ $S_n=\frac{n}{2}[2a+(n-1)d]$ $=\frac{n}{2}[2(9)+(n-1)4]$ $=\frac{n}{2}[14+4n]$ $=7n+2n^2$	✓ correct substitution ✓ simplification (2)
	Subtotal	**[6]**

QUESTION 3

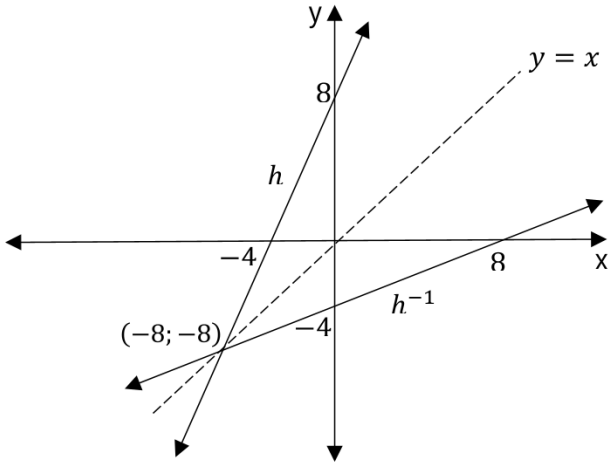
Q#	Suggested Solutions	Descriptors
3.1	$r = \frac{x+1}{3}$ $-1 < \frac{x+1}{3} < 1$ $-3 < x + 1 < 3$ $-4 < x < 2$, but $x \neq -1$	✓ $-1 < \frac{x+1}{3} < 1$ ✓ $-4 < x < 2$ ✓ $x \neq -1$ (3)
3.2	$S_\infty = \frac{a}{1-r}$ $= \frac{\frac{5(1+1)}{3}}{1-\frac{1+1}{3}}$ $= 10$	✓ correct substitution ✓ answer (2)
	Subtotal	**[5]**

QUESTION 4

Q#	Suggested Solutions	Descriptors
4.1	$(2x - 5) - (x + 2) = (x + 2) - (1 - x)$ $x - 7 = 2x + 1$ $-8 = x$	✓ method ✓ simplification ✓ $x = -8$ (3)
4.2	$9\ ;\ -6\ ;\ -21$ $a = 9, d = -15, n = 100$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{100} = \frac{100}{2}[2(9) + (100-1)(-15)]$ $= -73\,350$	✓ correct substitution ✓ answer (2)
4.3	$\sum_{n=0}^{m-1}(24 - 15n) = S_{100} + 73\,320$ $= -73\,350 + 73\,320$ $\frac{m}{2}[2(24) + (m-1)(-15)] = -30$ $m(48 - 15m + 15) = -60$ $63m - 15m^2 = -60$ $0 = 15m^2 - 63m - 60$ $0 = 5m^2 - 21m - 20$ $0 = (5m + 4)(m - 5)$ $m = 5$ or $m \neq -\frac{4}{5}$	✓ RHS $= -30$ ✓ $\frac{m}{2}[2(24) + (m-1)(-15)]$ ✓ standard form ✓ $m = 5$ and rejection (4)
	Subtotal	**[9]**

QUESTION 5

Q#	Suggested Solutions	Descriptors
5.1	$p = 3$ $q = -1$	✓ $p = 3$ ✓ $q = -1$ (2)
5.2	$g(x) = -x + k$ $-1 = -(-3) + k$ ... at point $(-3; -1)$ $-4 = k$	✓ correct substitution ✓ $-4 = k$ (2)

5.3	$h(x) = -2(-x-4)$ $= 2x + 8$ For inverse: $x = 2y + 8$ $x - 8 = 2y$ $\frac{x-8}{2} = y$ $\therefore y = \frac{x}{2} - 4 \quad \Rightarrow h^{-1}(x) = \frac{x}{2} - 4$	✓ $h(x) = 2x + 8$ ✓ swapping x and y ✓ $y = \frac{x}{2} - 4$ (3)
5.4		✓ correct graph shapes ✓ x- and y-intercepts interchanging ✓ correct point of intersection ✓ axis of symmetry passes through the correct point of intersection (4)
	Subtotal	**[11]**

QUESTION 6

Q#	Suggested Solutions		Descriptors
6.1	$f(x) = ax^2$ $2 = a(-1)^2$ $\therefore a = 2$	$g(x) = b^x$ $2 = b^{-1}$ $b = \frac{1}{2}$	✓ $a = 2$ ✓ $b = \frac{1}{2}$ (2)
6.2	$x \leq 0$ or $x \geq 0$		✓ any one of the correct conditions (1)
6.3	$x \leq -1$ OR $x \in (-\infty; -1]$ OR $-\infty < x \leq -1$		✓ correct critical value ✓ correct notation (2)
6.4.1	$A'(-4; 2)$		✓ $(-4; 2)$ (1)
6.4.2	$A''(2; -4)$		✓✓ $(2; -4)$ (2)
		Subtotal	**[8]**

TOTAL = 50 MARKS

QUESTION 1

1.1 Solve for x.

1.1.1 $x(2x-3)=0$ (2)

1.1.2 $0=3x^2-5x-11$ (3)

1.1.3 $x(3x+4)-2(3x+4)\leq 0$ (3)

1.2 The roots of a quadratic equation, in terms of p, are given as:

$$x=\frac{4\pm\sqrt{8-p^3}}{p}$$

Determine the value(s) of p for the roots to be real. (3)

[11]

QUESTION 2

The first four (4) terms of a quadratic pattern are: $11\ ;\ 20\ ;\ 33\ ;\ 50\ ;\ \ldots$

2.1 Determine the general term of this pattern in the form $T_n=an^2+bn+c$. (4)

2.2 Prove that the sum of the first n first-differences of this quadratic pattern can be given by $S_n=2n^2+7n$. (2)

[6]

QUESTION 3

A convergent geometric series is given by: $\frac{5(x+1)}{3}+\frac{5(x+1)^2}{9}+\frac{5(x+1)^3}{27}+\ldots$

3.1 Calculate the values of x. (3)

3.2 If $x = 1$, calculate the sum to infinity, S_∞. (2)

[5]

QUESTION 4

$(1 - x)$; $(x + 2)$ and $(2x - 5)$ are the first three (3) terms of an arithmetic sequence.

4.1 Determine the value of x. (3)

4.2 If the first three (3) terms of this pattern are: 9 ; -6 ; -21 ; ... , calculate the numerical value of the sum of the first 100 terms, S_{100}. (2)

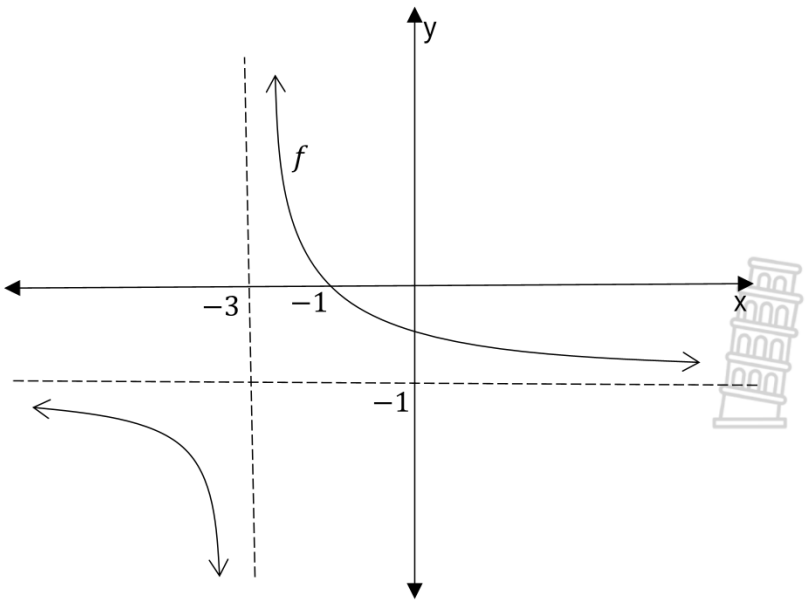
4.3 Hence, or otherwise, calculate m if :

$$\sum_{n=0}^{m-1}(24 - 15n) = S_{100} + 73\,320 \quad (4)$$

[9]

QUESTION 5

The graph of $f(x) = \frac{2}{x+p} + q$ is sketched below:



5.1 Write down the values of p and q (2)

5.2 The straight line $g(x) = -x + k$ is one of the axes of symmetry of the graph of f. Determine the value of k. (2)

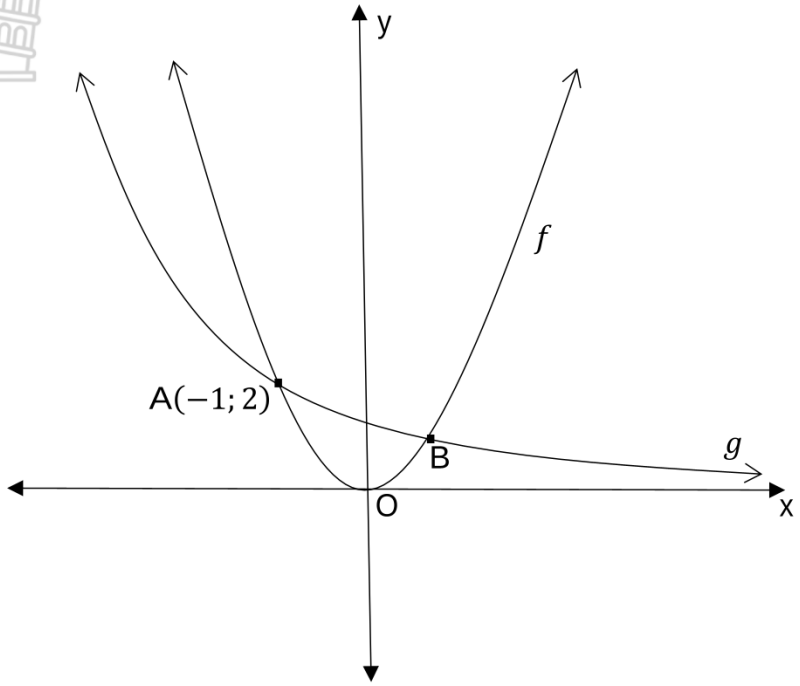
5.3 If $h(x) = -2[g(x)]$, determine the equation of the inverse of h, h^{-1}, in the form $h^{-1}(x) = mx + c$. (3)

5.4 Draw a neat sketch of the graphs of h and h^{-1} on the same set of axes. Clearly show all intercepts with axes, point of intersection and the axis of symmetry. (4)

[11]

QUESTION 6

The graphs of $f(x) = ax^2$ and $g(x) = b^x$ are sketched on the same set of axes. Points $A(-1; 2)$ and B are points of intersection of f and g. The graph of f has the turning point at the origin:



6.1 Calculate the values of a and b. (2)

6.2 The inverse of f is NOT a function. Write down at least one condition which can be used to restrict the domain of f such that its inverse will be a function. (1)

6.3 For which value(s) of x, where $x \in (-\infty; 0]$, will $g(x) \leq f(x)$? (2)

6.4 If $h(x) = g(x + 3)$, write down the coordinates of …

6.4.1 A', the new coordinates of A on the graph of h. (1)

6.4.2 A'', the new coordinates of A on the graph of h^{-1}, the inverse of h. (2)

[8]

TOTAL = 50 MARKS

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \;; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum f\dot{x}}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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