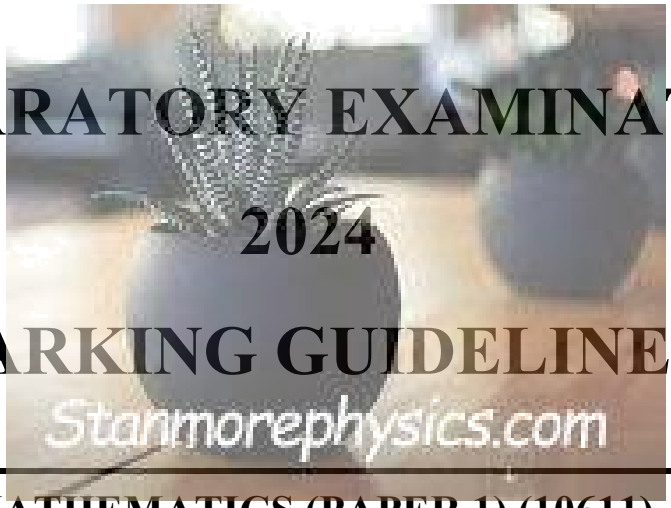
GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA



PREPARATORY EXAMINATION

2024

MARKING GUIDELINES

MATHEMATICS (PAPER 1) (10611)

22 pages

INSTRUCTIONS AND INFORMATION

A – Accuracy

CA – Continued Accuracy

S – Statement

R – Reason

S and R – Statement and Reason

NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed OUT an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies in ALL aspects of the marking guidelines.
- It is UNACCEPTABLE to assume values/answers in order to solve a question.

QUESTION 1

1.1	1.1.1	$2k = (x-5)(x-k)$ $\therefore 2k = (2-5)(2-k)$ $2k = (-3)(2-k)$ $2k = -6+3k$ $\therefore k = 6$	✓ simplification ✓ answer	(2)
	1.1.2	$2k = (x-5)(x-k)$ $\therefore 2(2) = (x-5)(x-2)$ $4 = x^2 - 7x + 10$ $\therefore 0 = x^2 - 7x + 6$ $0 = (x-1)(x-6)$ $\therefore x = 1$ or $x = 6$	✓ simplification ✓ standard form ✓ factors ✓ answers	(4)
1.2	1.2.1	$2x^2 + 3 = 8x$ $\therefore 2x^2 - 8x + 3 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\therefore x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)}$ $x = \frac{8 \pm \sqrt{64-24}}{4}$ $\therefore x = 3,58$ or $x = 0,42$ **NOTE:** Penalise 1 mark for rounding in this question ONLY. Candidates must show substitution to get full marks.	✓ standard form ✓ substitution into correct formula ✓✓ answers	(4)
	1.2.2	$\sqrt{2(x+10)} - 10 = x - 12$ $\sqrt{2(x+10)} = x - 2$ $2(x+10) = x^2 - 4x + 4$ $2x + 20 = x^2 - 4x + 4$ $0 = x^2 - 6x - 16$ $0 = (x+2)(x-8)$ $x \neq -2$ or $x = 8$	✓ isolate surd and square both sides ✓ standard form ✓ factors ✓ critical values with exclusion	(4)
	1.2.3	$3^x(x-5) < 0$ $3^x > 0$ for $x \in \mathbb{R}$ $x - 5 < 0$ $\therefore x < 5$	✓ $3^x > 0$ ✓ answer	(2)

1.3	$\sqrt{3^x}.9^y = 27 \qquad x+4y^2=6.....(2)$ $3^{\frac{x}{2}}.3^{2y} = 3^3$ $\frac{1}{2}x+2y=3$ $x+4y=6$ $x=6-4y....(1)$ sub (1) into (2) $6-4y+4y^2=6$ $4y^2-4y=0$ $4y(y-1)=0$ $y=0 \quad \text{or} \quad y=1$ $x=6 \qquad x=2$	✓ prime bases of 3 ✓ simplification (linear) ✓ expression for x ✓ factors ✓ both y-values ✓ both x-values	(6)
1.4	1.4.1 $2p+5=0$ $\therefore p=-\frac{5}{2}$	✓ answer	(1)
	1.4.2 $2p+5<0$ $\therefore p<-\frac{5}{2}$	✓ answer	(1)
			[24]

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QUESTION 2

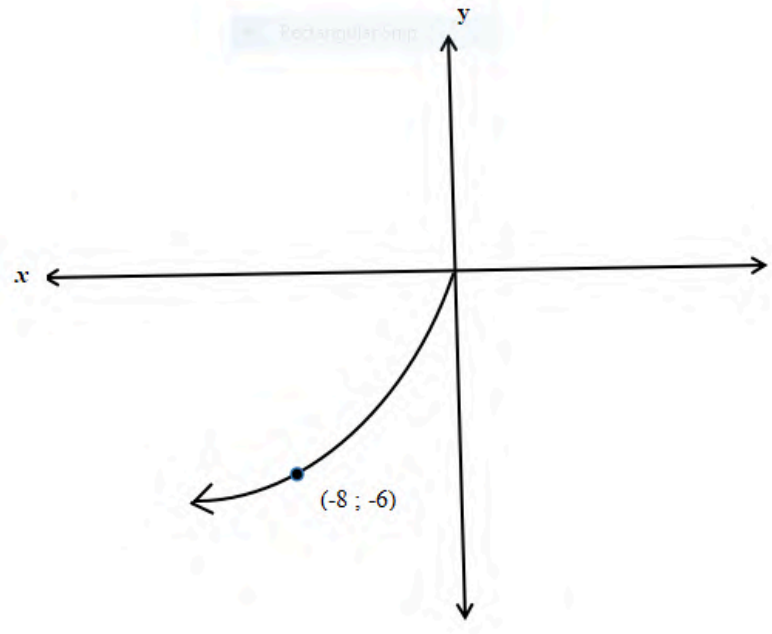
2.1	2.1.1	$T_4 = 27$	✓ answer	(1)
	2.1.2	0 5 14 27 44 5 9 13 17 1st differences 4 4 4 2nd differences $2a = 4$ $\therefore a = 2$ $3a + b = 5$ $3(2) + b = 5$ $\therefore b = -1$ $a + b + c = 0$ $2 - 1 + c = 0$ $\therefore c = -1$ $\therefore T_n = 2n^2 - n - 1$	✓ 2^{nd} differences ✓ value of a ✓ value of b ✓ value of c	(4)
	2.1.3	$860 = 2n^2 - n - 1$ $\therefore 0 = 2n^2 - n - 861$ $n = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-861)}}{2(2)}$ $\therefore n = 21$ or $n \neq -20,5$ There are 21 terms in the sequence. **NOTE:** Candidate must reject a negative answer or decimal answer to obtain full marks.	✓ equating correctly/or correct standard form ✓ substitution ✓ answer with rejection/ selection	(3)
2.2	Series: 110 + 121 +…… + 990 $\therefore a = 110$ and $d = 11$ $\therefore 110 + (n-1)11 = 990$ $(n-1)11 = 880$ $n - 1 = 80$ $\therefore n = 81$ $\therefore S_{81} = \frac{81}{2}[110 + 990]$ $\therefore S_{81} = 44\,550$ **OR**		✓ series with $T_1 = 110$ and $T_n = 990$ ✓ substitution into T_n formula ✓ value of n ✓ substitution into S_n formula ✓ answer	(5)

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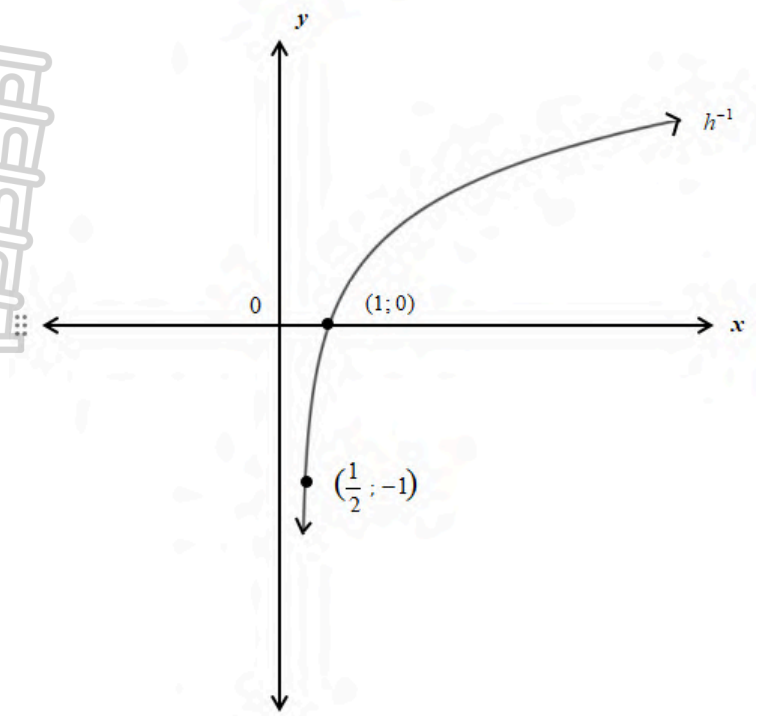
	$S_{81} = \frac{81}{2}[2(110) + (81-1)(11)]$ $S_{81} = 44550$	✓ substitution into S_n formula ✓ answer	
			[13]

QUESTION 3				
3.1	3.1.1	$r = \frac{4(x-2)^3}{8(x-2)^2}$ $\therefore r = \frac{x-2}{2}$ $-1 < \frac{x-2}{2} < 1$ $\therefore -2 < x-2 < 2$ $\therefore 0 < x < 4$	✓ expression for r ✓ $-1 < r < 1$ ✓ answer	(3)
	3.1.2	$a = 8(x-2)^2$ $a = 8(2{,}5-2)^2$ $\therefore a = 2$ $r = \frac{x-2}{2}$ $r = \frac{2{,}5-2}{2}$ $\therefore r = \frac{1}{4}$ $S_\infty = \frac{2}{1-\frac{1}{4}}$ $S_\infty = \frac{8}{3}$ **OR** $S_\infty = \frac{8(2{,}5-2)}{1-\left(\frac{2{,}5-2}{2}\right)}$ $S_\infty = \frac{8}{3}$	✓ value of a ✓ value of r ✓ substitution into correct formula ✓ answer ✓ value of a (numerator) ✓ value of r (denominator) ✓ substitution into correct formula ✓ answer	(4)
3.2	3.2.1	$n = (12-3)+1$ $\therefore n = 10$	✓ answer	(1)

	3.2.2	$a = 3(-2)^{3-2}$ $\therefore a = -6$ $r = -2$ $\therefore S_n = \frac{a(1-r^n)}{1-r}$ $\therefore S_n = \frac{-6(1-(-2)^{10})}{1+2}$ $\therefore S_n = 2046$	✓ value of a and r ✓ correct substitution in correct formula ✓ answer	(3)
				[11]

QUESTION 4			
4.1	$f(x)=ax^2 \quad x\le 0$ $-8=a(-6)^2$ $-8=36a$ $\therefore a=-\frac{2}{9}$ **NOTE**: Value of a must be in simplified form.	✓ substitute point P ✓ answer	(2)
4.2	$x=-\frac{2}{9}y^2 \quad ; \quad y\le 0$ $\therefore y^2=-\frac{9}{2}x \quad y\le 0$ $\therefore y=-\sqrt{-\frac{9}{2}x}$ **NOTE**: No penalty for not writing constraints of y.	✓ interchange x and y ✓ answer	(2)
4.3	$y\le 0$	✓ answer	(1)
4.4		✓ shape ✓ point on f^{-1}	(2)
4.5	$-y=-\sqrt{-\frac{9}{2}x}$ $\therefore y=\sqrt{-\frac{9}{2}x}$ **NOTE:** Answer only, award FULL marks.	✓ setting equation for reflection across $y=x$ and x-axis ✓ answer	(2)
			[9]

QUESTION 5				
5.1	$P(-\sqrt{3}\ ;2)$ **NOTE:** If a candidate draws a sketch indicating understanding of a rotation of 90^0 anticlockwise and shows the point in the second quadrant, award 1 mark. If a candidate has a negative x-coordinate, indicating understanding of the point being in the second quadrant, award 1 mark.		✓✓ answer	(2)
5.2	5.2.1	$a^0 = 1$ for $a \in \mathbb{R}$; $a \neq 0$ $\therefore h(0) = a^0$ $\therefore h(0) = 1$ $\therefore Q(0\ ;1)$ **NOTE:** Any other valid answer	✓ making $x = 0$ ✓ value of $h(0)$ (A)	(2)
	5.2.2	$h(x) = a^x$ $h(-1) = a^{-1} = \frac{1}{2}$ $\therefore \frac{1}{a} = \frac{1}{2}$ $\therefore a = 2$	✓ substitute point A ✓ answer	(2)
	5.2.3	h: $y = 2^x$ $\therefore h^{-1}$: $x = 2^y$ $\therefore y = \log_2 x$ **NOTE:** Answer only, award FULL marks. Accept an answer in terms of a: $y = \log_a x$	✓ interchange x and y ✓ answer	(2)

	5.2.4		✓ shape (increasing log function) ✓ x-int ✓ asymptote	(3)
	5.2.5	$\log_2 x > -1$ $\therefore x > 2^{-1}$ $\therefore x > \frac{1}{2}$ **NOTE**: Answer only, full marks	✓ log to exponential form ✓ answer	(2)
	5.2.6	$100.3^x = 2^x$ $100 = \frac{2^x}{3^x}$ $100 = \left(\frac{2}{3}\right)^x$ $\log_{\frac{2}{3}} 100 = x$ $x = -11,36$ **OR**	✓ simplification ✓ express correctly in terms of logs ✓ answer	(3)

		$h(x) = g(x)$ $\therefore 2^x = 100.3^x$ $\therefore \frac{2^x}{3^x} = 100$ $\therefore \left(\frac{2}{3}\right)^x = 100$ $\therefore \log\left(\frac{2}{3}\right)^x = \log 100$ $\therefore x \log\frac{2}{3} = \log 100$ $x = \frac{\log 100}{\log\frac{2}{3}}$ $\therefore x = -11,36$	✓ express correctly in terms of logs ✓ x as subject ✓ answer	
	5.3.1	$p = \log\left(10 + \frac{q}{2}\right)$ $p = \log\left(10 + \frac{1980}{2}\right)$ $p = \log(1000)$ $\therefore p = 3$ Total Price $R3 \times 1980$ $= R5940$	✓ $p = \log 1000$ ✓ value of p ✓ total price	(3)
	5.3.2	$p = \log\left(10 + \frac{q}{2}\right)$ $\therefore 2 = \log_{10}\left(10 + \frac{q}{2}\right)$ $\therefore 10^2 = 10 + \frac{q}{2}$ $\therefore 200 = 20 + q$ $\therefore q = 180$	✓ log to exponential equation ✓ answer	(2)
				[21]

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QUESTION 6			
6.1	$x \in \mathbb{R}$; $x \neq -1$ **NOTE**: Both conditions must be stated.	✓ answer	(1)
6.2	$x = -1$ $y = 2$ **NOTE**: Do not accept answers written in terms of p and q.	✓ answer ✓ answer	(2)
6.3	$-x + 4 = \frac{2}{x+1} + 2$ $\therefore (-x+4)(x+1) = 2 + 2(x+1)$ $\therefore -x^2 + 3x + 4 = 2 + 2x + 2$ $\therefore 0 = x^2 - x$ $\therefore 0 = x(x-1)$ $\therefore x = 0$ or $x = 1$ $\therefore x_D = 1$ $\therefore y_D = -1 + 4 = 3$ $\therefore$ D(1 ; 3) **NOTE:** Does not have to be in coordinate form.	✓ equating ✓ simplification ✓ standard form ✓ x-values with selection (A) ✓ y-value	(5)
			[8]

QUESTION 7				
7.1	$A = P(1+i)^n$ $\therefore 2x = x\left(1+\frac{i}{4}\right)^{24}$ $\therefore (1+\frac{i}{4})^{24} = 2$ $\therefore 1+\frac{i}{4} = \sqrt[24]{2}$ $\therefore \frac{i}{4} = 0{,}0293...$ $\therefore i = 0{,}1172...$ $\therefore$ Annual % interest rate = 11,72%		✓ substitution into correct formula ✓ value of i ✓ answer	(3)
	7.2.1	$A = P(1+i)^n$ $\therefore A = 10000(1+0{,}0079...)^5$ $\therefore A = 10000(1{,}0079...)^5$ $\therefore A =$ R10 402,15	✓ correct i and n ✓ substitution into correct formula ✓ answer	(3)
	7.2.2.	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $\therefore \frac{450[1-(1{,}0079)^{-n}]}{0{,}0079} = 10402{,}15$ $\therefore [1-(1{,}0079)^{-n}] = 0{,}183$ $\therefore 1-0{,}183 = (1{,}0079)^{-n}$ $\therefore 0{,}8169 = (1{,}0079)^{-n}$ $\therefore -n = \log_{1{,}0079} 0{,}8169$ $\therefore -n = -25{,}63...$ $\therefore n = 26$ **NOTE**: Answer must be a natural number.	✓ substitution into correct formula ✓ $[1-(1{,}0079)^{-n}] = 0{,}183$ ✓ correct use of logs ✓ answer	(4)
	7.2.3	Balance of the loan after the 25^{th} payment: = value of loan - value of annuity at that time $= 10\ 402{,}15(1{,}0079)^{25} - \frac{450[(1{,}0079)^{25}-1]}{0{,}0079}$ $= 12\ 668{,}89 - 12\ 386{,}53$ = R282,36 **OR**	✓ $10402{,}15(1{,}0079)^{25}$ ✓ $\frac{450[(1{,}0079)^{25}-1]}{0{,}0079}$ ✓ answer	(3)

		$Pv = \dfrac{450\left[1-\left(1+\dfrac{0.095}{12}\right)^{-0,6315128}\right]}{\dfrac{0.095}{12}}$ $Pv = R282,36$	✓ correct substitution in Pv formula ✓ $n = 0,6315128$ ✓ answer	
				[13]

QUESTION 8			
8.1	$f(x) = -2x^2 + 3x$ $f(x+h) = -2(x+h)^2 + 3(x+h)$ $f(x+h) = -2(x^2 + 2xh + h^2) + 3x + 3h$ $f(x+h) = -2x^2 - 4xh - 2h^2 + 3x + 3h$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \to 0} \frac{-2x^2 - 4xh - 2h^2 + 3x + 3h - (-2x^2 + 3x)}{h}$ $f'(x) = \lim_{h \to 0} \frac{-2x^2 - 4xh - 2h^2 + 3x + 3h + 2x^2 - 3x}{h}$ $f'(x) = \lim_{h \to 0} \frac{-4xh - 2h^2 + 3h}{h}$ $f'(x) = \lim_{h \to 0} \frac{h(-4x - 2h + 3)}{h}$ $f'(x) = \lim_{h \to 0} (-4x - 2h + 3)$ $f'(x) = -4x + 3$ **NOTE:** Penalise ONE mark for incorrect notation in this question only.	✓ substitution ✓ simplification ✓ correct factorisation ✓ answer	(4)
8.2	$f(x) = \frac{3x^2}{2} - 24\sqrt{x}$ $\therefore f(x) = \frac{3x^2}{2} - 24x^{\frac{1}{2}}$ $\therefore f'(x) = 3x - \frac{12}{x^{\frac{1}{2}}}$ $\therefore f'(9) = 3(9) - \frac{12}{9^{\frac{1}{2}}}$ $\therefore f'(9) = 23$	✓ simplification ✓✓ derivatives ✓ substitution ✓ answer	(5)

8.3	Minimum value at $x = 4 \quad \therefore g'(x) = 0$ at $x = 4$	✓ $\therefore g'(4) = 0$	
	$g(x) = ax^2 + bx^{-1}$ $\therefore g'(x) = 2ax - bx^{-2}$	✓ derivative	
	$\therefore 2a(4) - b(4)^{-2} = 0$ $\therefore 8a - \frac{b}{16} = 0$ $\therefore 128a - b = 0 \cdots\cdots (1)$	✓ equation 1	
	Function value at $x = 4$ is 96 $\therefore g(4) = 96$ $\therefore a(4)^2 + b(4)^{-1} = 96$ $\therefore 16a + \frac{b}{4} = 96$ $\therefore 64a + b = 384 \cdots\cdots (2)$	✓ equation 2	
	$(1) + (2)$ $\therefore 192a = 384$ $\therefore a = 2$	✓ value of a	
	$(1) \cdots\cdots \therefore b = 256$	✓ value of b	(6)
			[15]

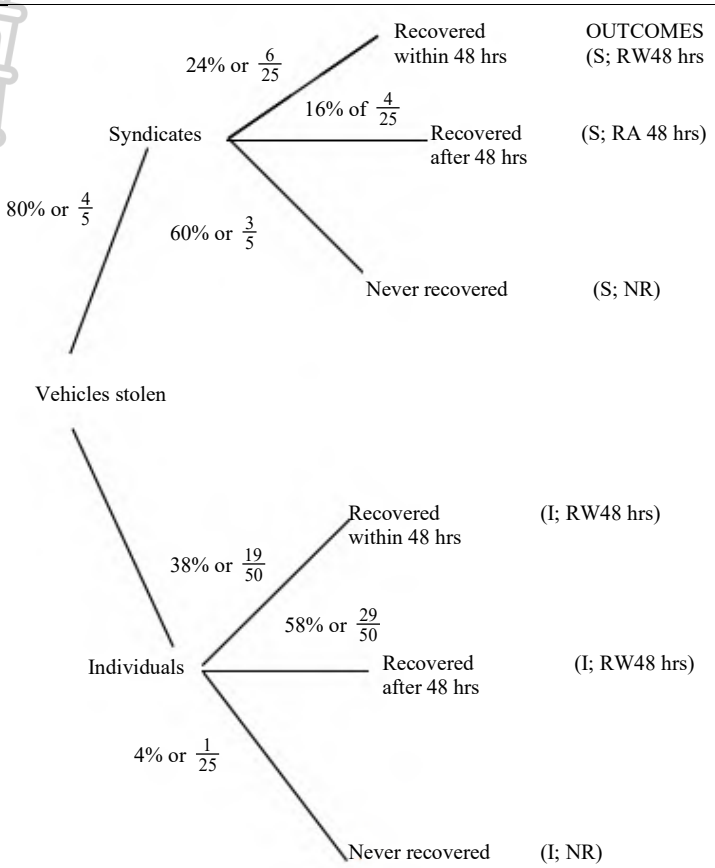
QUESTION 9				
9.1	9.1.1	E(0 ; −4)	✓ answer	(1)
	9.1.2	$y = a(x+2)(x-6)$ roots are: -2 and 6 $-4 = a(2)(-6)$ substitute point (0 ; −4) $\therefore 12a = 4$ $\therefore a = \frac{1}{3}$ equation of g': $y = \frac{1}{3}(x+2)(x-6)$ $y = \frac{1}{3}(x^2 - 4x - 12)$ $\therefore y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$	✓ substitute roots and point ✓ value of a ✓ substitute a and simplification ✓ answer	(4)
	9.1.3	At the turning point of g: $g'(x) = 0$ $\therefore x = -2$ and $x = 6$	✓✓ answers	(2)
	9.1.4	$x = 2$	✓ answer	(1)
	9.1.5	For $x < -2$, $g'(x) > 0$ $\therefore$ the gradient of g is positive and for the values of x immediately right of -2. For $g'(x) < 0$, the gradient of g is negative. **NOTE:** Any other valid explanation that indicates understanding	✓✓ explanation	(2)
9.2	The gradient of the tangent to h at any value of x is: $h'(x) = 12x^2 + 5$ $x^2 \geq 0$ for all $x \in \mathbb{R}$ $\therefore 12x^2 \geq 0$ for all $x \in \mathbb{R}$ $\therefore 12x^2 + 5 > 0$ for all $x \in \mathbb{R}$ (the gradient is ≥ 5) $\therefore$ The gradient of the tangent is always positive. It is impossible to draw a tangent to h which has a negative gradient. **OR** A negative gradient requires: $12x^2 + 5 < 0$ $\therefore 12x^2 < -5$ $\therefore x^2 < -\frac{5}{12}$ This is impossible, a square is always ≥ 0		✓ derivative ✓ explanation ✓ value of x^2 ✓ explanation	(2)
				[12]

QUESTION 10			
10.1	$FE = FA = y - x$ $AB + BC + CD + DE = 2x + 2y$ $\therefore 2x + 2y = 30$ $\therefore 2y = 30 - 2x$ $\therefore y = 15 - x$ $A(x) = y^2 - (y - x)^2$ $= y^2 - y^2 + 2xy - x^2$ $= 2xy - x^2$ $= 2x(15 - x) - x^2$ $= 30x - 2x^2 - x^2$ $= 30x - 3x^2$ For maximum: $A'(x) = 0$ $\therefore 30 - 6x = 0$ $\therefore x = 5\,m$ $\therefore y = 15 - 5$ $\therefore y = 10\,m$	✓ expression for FE and FA in terms of x and y ✓ expression for y ✓ substitute into area formula ✓ substitute for y ✓ expression for area ✓ derivative = 0 ✓ value of x ✓ value of y	
			[8]

QUESTION 11			
11.1	For mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$ $0,7 = 0,4 + k$ $\therefore k = 0,3$ **NOTE:** Answer only, full marks. If a candidate writes down: $1 - 0,7 = 0,3$; award $\frac{0}{2}$	✓ substitution ✓ answer	(2)
11.2	For independent events: $P(A \text{ and } B) = P(A) \times P(B)$ $= 0,4k$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,7 = 0,4 + k - 0,4k$ $0,3 = 0,6k$ $\therefore k = 0,5$ **NOTE**: Answer only, $\frac{1}{3}$ Wrong formulas, $\frac{0}{3}$	✓ $P(A \text{ and } B) = 0,4k$ ✓ correct substitution for P(A or B) ✓ answer	(3)
			[5]

QUESTION 12

12.1 12.1.1



NOTE: S (Syndicates)
I (Individuals)
RW (Recovered within 48 hours)
RA (Recovered after 48 hours)
NR (Never recovered)

Penalty of 1 mark for a mistake in the 2^{nd} branch level.

No values but correct tree diagram, award $\frac{2}{3}$

Outcomes need NOT be shown.

✓ first level

✓ second level, (syndicates branch, labels must be on.)

✓ second level. (individuals branch labels must be on.)

(3)

12.1.2

$$P(S\ ;\ RW48hrs) = \frac{80}{100} \times \frac{24}{100}$$

$$= \frac{1920}{10000}$$

$$= 0,192$$

$$= 19,2\%$$

✓ probability

✓ answer

		OR P(S ; RW48hrs) $= \frac{4}{5} \times \frac{6}{25}$ $= \frac{24}{125}$ **NOTE:** Answer Only, award FULL marks if 12.1.2 is accurately drawn. Penalty of 1 mark for writing to 1 decimal place. Accept: 0,19 and 0,192	**OR** ✓ probability ✓ answer	(2)
	12.1.3	P(stolen and not recovered) $= \left(\frac{80}{100} \times \frac{60}{100}\right) + \left(\frac{20}{100} \times \frac{4}{100}\right)$ $= 0,488$ $= 48,8\%$ **OR** P(stolen and not recovered) $= \left(\frac{4}{5} \times \frac{3}{5}\right) + \left(\frac{1}{5} \times \frac{1}{25}\right)$ $= \frac{12}{25} + \frac{1}{125}$ $= \frac{61}{125}$ **NOTE:** Accept 0,49	✓ P(Stolen) ✓ P(Not recovered) ✓ answer	(3)
12.2	$9 \times 9 \times 9 \times 5 \times 4$ $= 14580$		✓✓ combination (A) ✓ answer	(3)
				[11]

TOTAL: 150