education

Department of Education
FREE STATE PROVINCE

GRADE 12



MARKS: 150

DURATION: 3 HOURS

This question paper consists of 11 pages,1 information sheet and 1diagram sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. You may use an approved scientific calculator (non-programmable and non-graphical), unless otherwise stated.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise
6. A diagram sheet is attached at the end of the question paper.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.
9. Answers only will NOT necessarily be awarded full marks.

QUESTION 1

1.1 Solve for x:

1.1.1 $(x+2)(x-5)=0$ (2)

1.1.2 $x(2x+3)=3$ (correct to TWO decimal places) (5)

1.1.3 $(x-1)(2-x)\geq 0$ (3)

1.1.4 $3\sqrt{x-1}=1-x$ (5)

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1.2 Consider: $\left(\frac{1}{81}\right)^{-x}=9^{y+3}$ and $y^2+x^2-3x=-1$

1.2.1 Show that $y=2x-3$ (2)

1.2.2 Solve for x and y simultaneously (5)

1.3 Show that the quadratic equation $x^2+px^2+2px+p=1$ has two distinct real roots for all real values of the constant p, except for one value which must be stated. (3)

[25]

QUESTION 2

2.1 The first three terms of a linear number pattern are:

$(m+1);(m^2+m);(3m^2-m-4)$

2.1.1 Determine the value(s) of m. (3)

2.1.2 Determine the value of the first three terms, when $m>0$. (2)

2.2 The pattern: $1;-5;-13;-23.....$ is such that the second difference is constant.

2.2.1 Determine the 5^{th} number in the pattern. (1)

2.2.2 Determine the general term of the pattern. (4)

2.2.3 Which term of the pattern has a value of -299? (3)

2.2.4 Determine the maximum value of the pattern. (2)

[15]

QUESTION 3

3.1 Given a geometric sequence: ___; 6 ; 12 ; 24 ; 48 ; .....

3.1.1 Determine the common ratio. (1)

3.1.2 Determine the value of the first term. (1)

3.1.3 Determine T_n. (2)

3.2 If the numbers 42, 32 and 2 are added to the first, second and third terms of a geometric sequence respectively, the three terms will all be equal.

Calculate the values for the three terms. (4)

3.3 The first term of an arithmetic sequence is 51 and the eighth term is 100.

3.3.1 Determine the constant difference. (2)

3.3.2 Find the twentieth term of the series. (2)

3.4 Calculate the sum of the multiples of 7 between 1 and 1000. (4)

[16]

QUESTION 4

4.1 Given the geometric series: $81x^2 + 27x^3 + 9x^4 + \ ...$

4.1.1 For which value(s) of x will the series converge? (3)

4.1.2 Calculate S_∞, sum to infinity if $x = \frac{1}{2}$. (3)

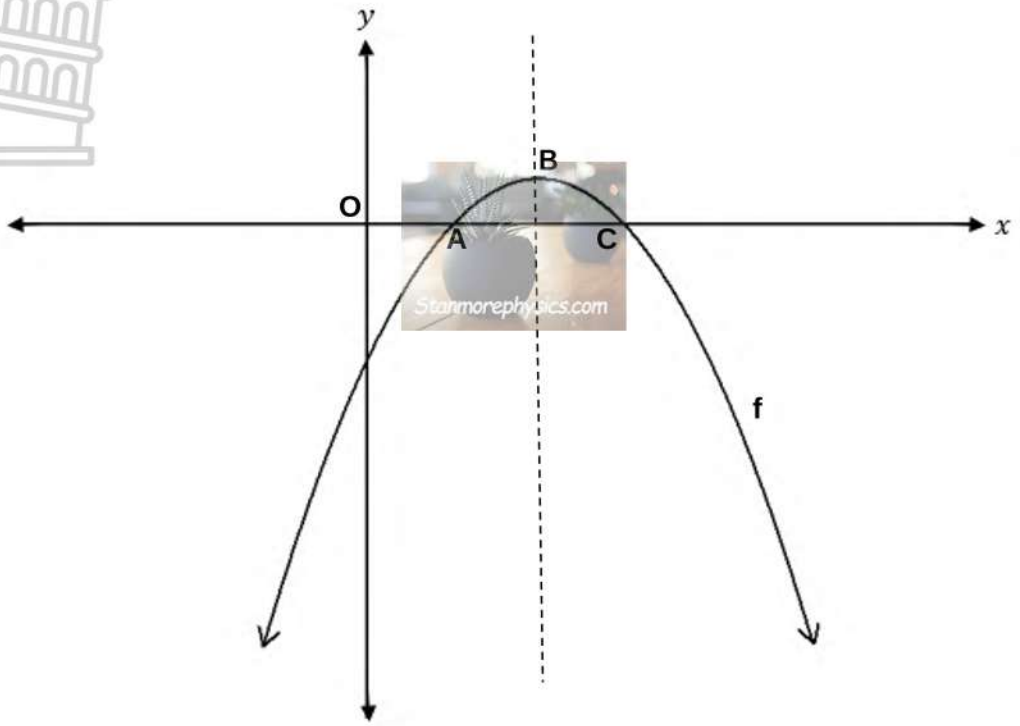
4.2 Given : $80 = \sum_{n=1}^{20} (25 + np)$

Find the value of the constant p. (4)

[10]

QUESTION 5

5.1 Sketched below is the parabola f, with the equation $f(x) = -(x-2)^2 + 1$.



5.1 Write down the coordinates of B. (1)

5.2 Write down the equation of the axis of symmetry. (1)

5.3 Determine the coordinates of A and C. (3)

5.4 For which value(s) of x will $f(x) \leq 0$. (2)

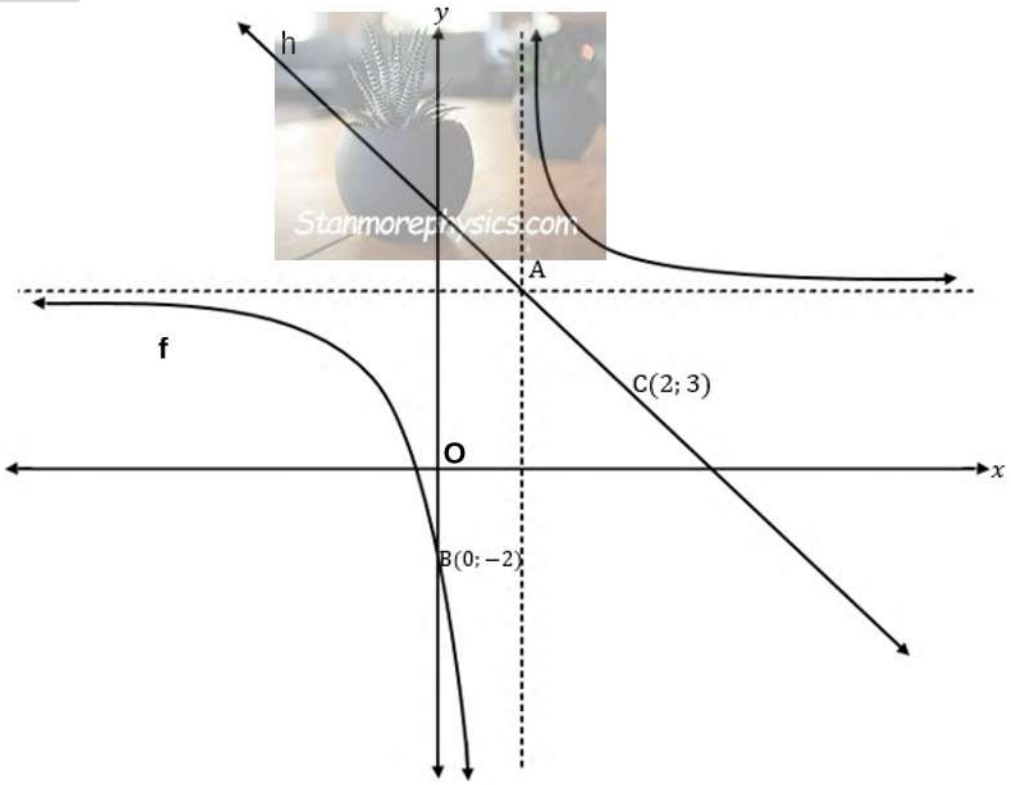
5.5 Determine the average gradient between A and B. (2)

5.6 The graph of $g(x)$ is obtained by shifting the graph of f, 2 units to right, 1 unit downwards, write down the equation of $g(x)$, and then sketch the graph of $g(x)$ on the same set of axis. (3)

[12]

QUESTION 6

In the sketch below the graph of $f(x)=\frac{a}{x+p}+4$ is given. The asymptotes of f intersect at point A. The graph of f cuts the y-intercept at $B(0;-2)$. The axis of symmetry of f, is the line h. Point C coordinates $C(2;3)$ is the point on h.



6.1.1 Determine the equation of h. (2)

6.1.2 Determine the coordinates of point A. (2)

6.1.3 Determine the equation of f. (3)

6.1.4 Determine the equation of the asymptotes of $f(x+1)$ (2)

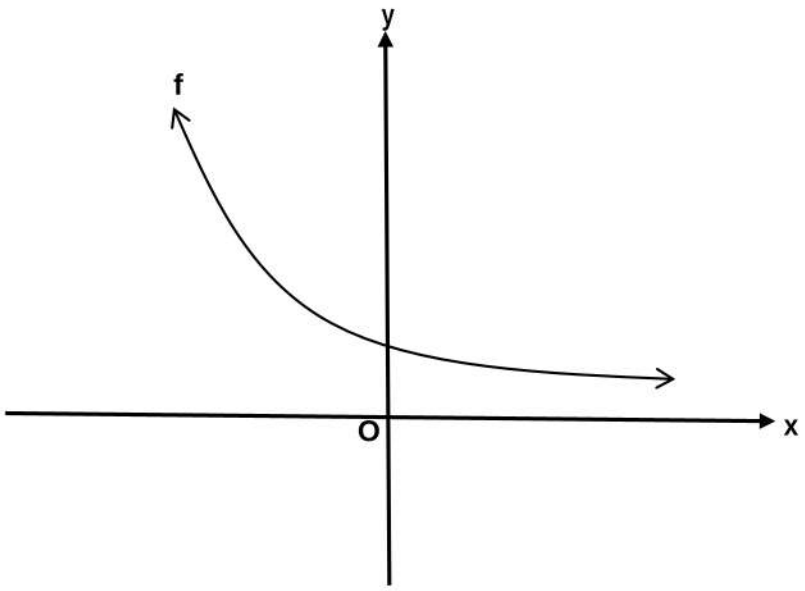
6.1.5 Write down the coordinates of the image of $D\left(-\frac{1}{2};0\right)$ if D is reflected about the axis of symmetry $y=x+3$ (2)

6.2 Draw a rough sketch of the graph $y=a-\frac{1}{b-x}, (a>0, b<0)$ (3)

[14]

QUESTION 7

Sketched below is the graph of $f(x) = k^x; k > 0$. The point $\left(2; \frac{1}{9}\right)$ lies on f.



7.1 Determine the value of k. (2)

7.2 Write down the range of f. (1)

7.3 Explain the transformation of f to f^{-1}. (1)

7.4 Determine the equation of f^{-1} in the form $y = ...$ (2)

7.5 Sketch the graph of f^{-1}. Indicate on your graph the coordinates of ONE point. (3)

7.6 Prove that $\left[f(x)\right]^2 - \left[f(-x)\right]^2 = f(2x) - f(-2x)$. (3)

[12]

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QUESTION 8

8.1 Given: $f(x) = x^2 - 3$

Use first principles to find $f'(x)$. (5)

8.2 Determine $\frac{dy}{dx}$ if:

$$y = \frac{9x^4 - 6}{3x}$$ (3)

8.3 Evaluate leave your answer in a surd form.

$$\frac{d}{dx}\left[\frac{\sqrt[3]{x^3} - 2x\sqrt{x}}{3x}\right]$$ (4)

8.4 Given $f(x) = x^3 - 2x + 1$ and the gradient of the tangent at the point of contact is 3 determine the x-coordinate(s) at the point of contact. (2)

[14]

QUESTION 9

9. A cubic function has the equation $f(x) = -x^3 + 5x^2 - 7x + 3$.

9.1.1 Determine the coordinates of y-intercept. (1)

9.1.2 Determine the x-intercepts. (4)

9.1.3 Determine the coordinates of the turning point. (4)

9.1.4 Sketch the graph of f. (4)

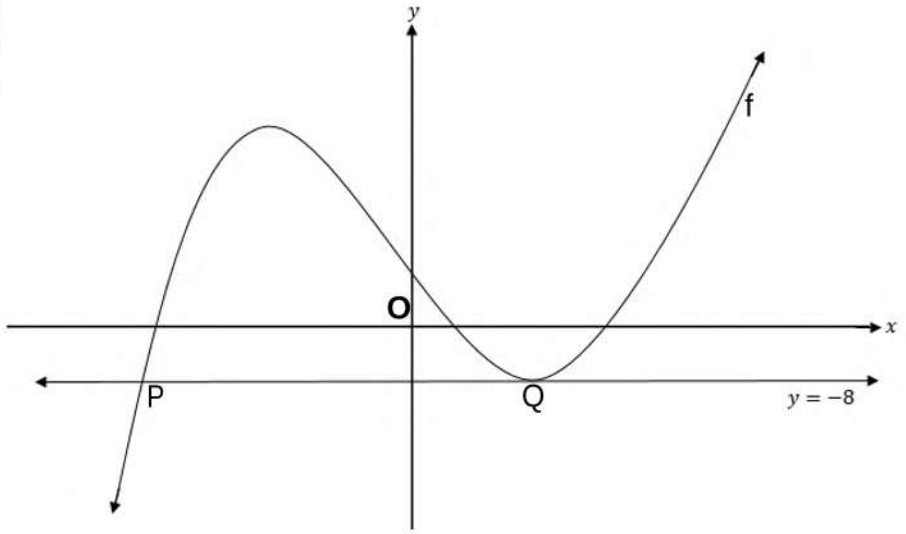
9.1.5 Determine the x-value of point of the inflection. (3)

9.1.6 Sketch the graph of $f''(x)$ on the same set of axes. (2)

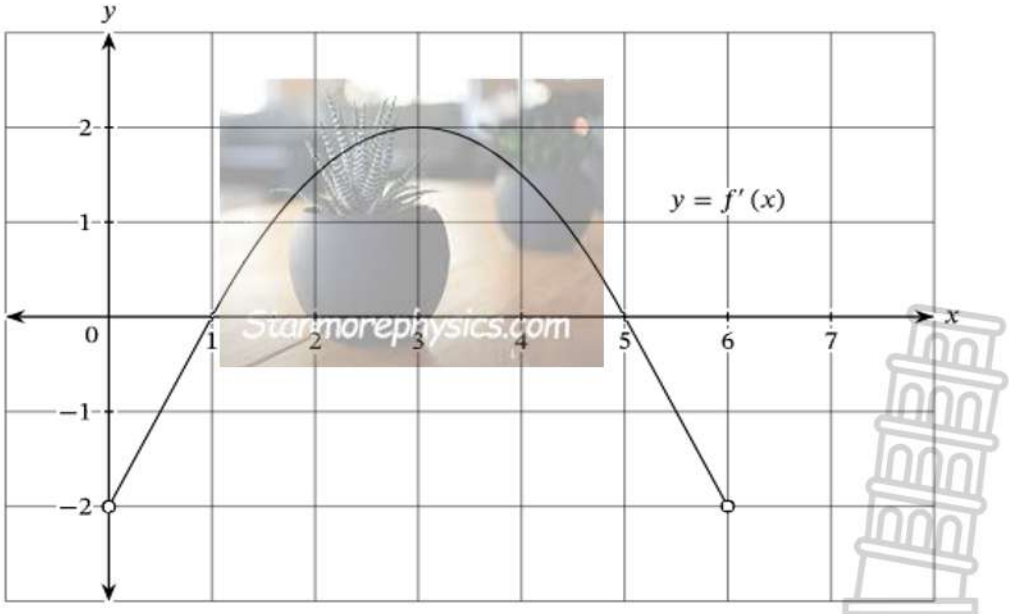
9.1.7 Write down the value(s) of x for which the graph of f is concave up. (2)

9.1.8 For which value(s) of x will $f'(x).f(x) \geq 0$? (2)

9.2 The graph of $f(x) = x^3 + 3x^2 + 24x + 20$ is shown below. The straight line with equation $y = -8$ touches the graph of $f(x)$ at the turning point $Q(2;-8)$ and crosses the graph of $f(x)$ at point P, as shown in the figure below. Determine the coordinates of P. (4)



9.3 The graph of the derivative f' of a function f is shown.



9.3.1 Determine the x values at the turning points of the graph f. (2)

9.3.2 On what intervals is f decreasing? (4)

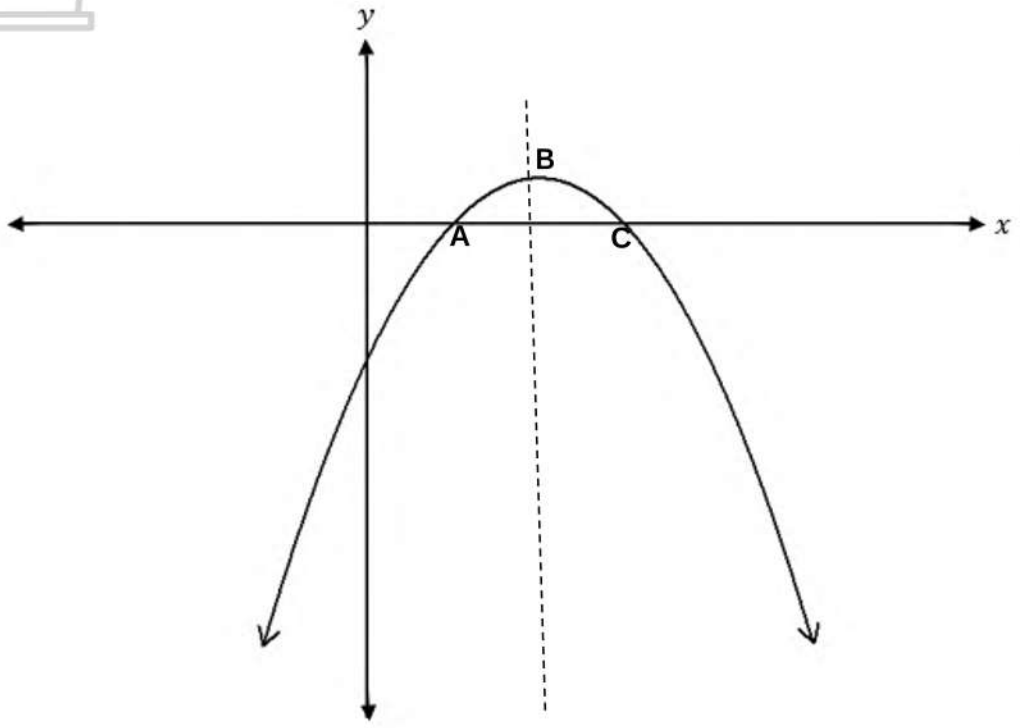
[32]

TOTAL: 150

DIAGRAM SHEET 1

QUESTION 5.7

NAME: _____



INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1};\ r \neq 1 \qquad S_\infty = \frac{a}{1-r};\ -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta\text{ABC} = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad \mathrm{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$