Province of the
EASTERN CAPE
EDUCATION

# NATIONAL SENIOR CERTIFICATE

## GRADE 12

## JUNE 2023

# MATHEMATICS P1

**MARKS:** 150

**TIME:** 3 hours

Stanmorephysics

*JMATHE1*

This question paper consists of 9 pages, including an information sheet.

## INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
3. Answers only will not necessarily be awarded full marks.
4. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. An information sheet, with formulae, is included at the end of the question paper.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

## QUESTION 1

1.1 Solve for $x$:

1.1.1 $x^2 - 9 = 0$ (2)

1.1.2 $x - 5 + \frac{2}{x} = 0$ (correct to TWO decimal places) (4)

1.1.3 $x = 1 + \sqrt{7 - x}$ (5)

1.1.4 $x^2 + 2x - 15 \geq 0$ (3)

1.2 Solve simultaneously for $x$ and $y$ in:

$$y + 2x = 3$$
$$y^2 - y = 3x^2 - 5x$$ (6)

1.3 Simplify completely, WITHOUT the use of a calculator: $\sqrt[n]{\frac{10^n + 2^{n+2}}{5^{2n} + 4(5^n)}}$ (4)

**[24]**

## QUESTION 2

2.1 Given the following geometric series: $\frac{24}{x} + 12 + 6x + 3x^2 + \ldots$

2.1.1 Determine the value of $r$, the common ratio, in terms of $x$. (1)

2.1.2 Determine the values of $x$ for which this series converges. (2)

2.1.3 If $x = 4$, determine the sum of the series to 15 terms. (3)

2.2 Calculate: $\sum_{n=1}^{\infty} 6(2)^{-n}$ (3)

2.3 The sum of the first $n$ terms of an arithmetic series is given by $S_n = -n^2 + 8n$.

2.3.1 Calculate the sum of the first 15 terms. (2)

2.3.2 Calculate the value of $T_{15}$. (2)

2.3.3 If the first term of the series is 7, which term of the series will have a value of $-169$? (4)

**[17]**

## QUESTION 3

Consider the following quadratic number pattern: 95 ; 72 ; $y$ ; 32 ; . . .

3.1 Determine the value of $y$. (2)

3.2 If $y = 51$, determine the general term of the number pattern in the form $T_n = an^2 + bn + c$. (4)

3.3 Determine $T_{22}$. (1)

3.4 Which term in the number pattern will be equal to 1 040? (4)

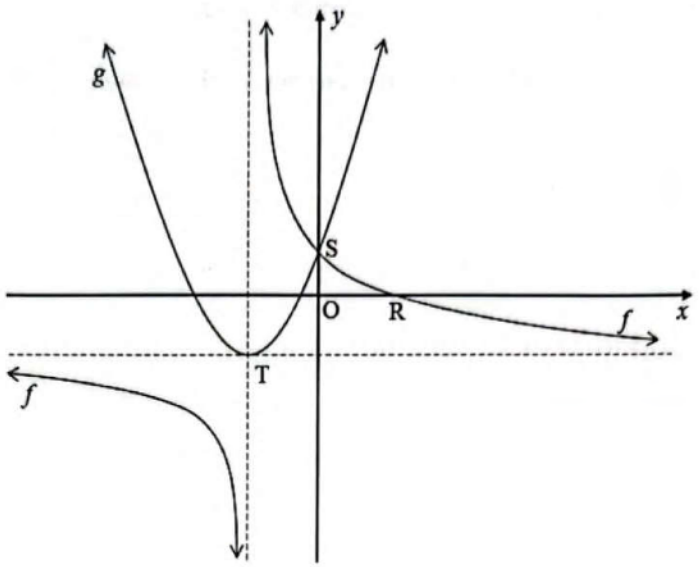
**[11]**

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## QUESTION 4

The diagram below shows the graphs of $f(x)=\frac{5}{x+p}+q$ and $g(x)=5x^2+10x+3$.

The two graphs intersect at S, the $y$-intercept of both graphs. R is the $x$-intercept of $f$.
The asymptotes of $f$ cut at T, the turning point of $g$.




4.1 Write down the coordinates of S. (2)

4.2 Determine:

4.2.1 The coordinates of T (4)

4.2.2 The values of $p$ and $q$ (2)

4.2.3 The length of OR (2)

4.2.4 The range of $g$ (2)

4.3 Determine the equation of:

4.3.1 The tangent to $g$ at S (3)

4.3.2 The axis of symmetry of $f$, with a positive gradient (2)

4.4 For which values of $x$ will $g'(x).f(x)\leq 0$? (2)
**[19]**



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## QUESTION 5

Given: $h(x) = a^x$; $a > 0$ and $a \neq 1$. B$\left(-1 ; \frac{1}{2}\right)$ is a point that lies on $h$, the graph of $h(x)$.

5.1 Determine the value of $a$. (2)

5.2 Write down the equation of $h^{-1}$ in the form $y = \ldots$ (2)

5.3 Sketch the graphs of $h$ and $h^{-1}$ on the same set of axes. Clearly show all intercepts with the axes. (4)

5.4 Write down the domain of $h^{-1}$. (1)

5.5 Determine the value(s) of $x$ for which $h^{-1}(x) > 1$. (1)

5.6 If it is given that $t(x) = \left(\frac{1}{2}\right)^x - 1$.

5.6.1 Describe the transformation from $h$ to $t$. (2)

5.6.2 Determine the equation of the asymptote of $t$. (1)

**[13]**

## QUESTION 6

6.1 A school bought computers for R980 000. The value of the computers depreciates annually at a rate of 9,2% p.a. on the reducing-balance method. Calculate the book value of the computers after 7 years. (3)

6.2 Siphokazi invests R13 500 for a certain number of years. She earns interest at a rate of 8,2% per annum, compounded annually. The final value of the investment is worth R20 020,28.
For how many years was the money invested? (4)

6.3 On 1 January 2017 Nelson deposited R3 500 into a savings account. On 1 January 2020, he deposited another R5 700 into the same account. The interest rate for the first two years (starting from 1 January 2017) is 7% per annum compounded quarterly, and the interest rate for the last three years is 8% per annum compounded monthly. Calculate the amount in the savings account after 5 years. (6)

**[13]**

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## QUESTION 7

7.1 Determine $f'(x)$, from first principles, if $f(x)=5-2x^2$. (4)

7.2 Determine:

7.2.1 $f'(x)$, if $f(x)=2x^5-7\sqrt{x}+\frac{1}{x}$ (4)

7.2.2 $\frac{d}{dx}\left[\frac{2x^2-x-6}{2x+3}\right]$ (3)

**[11]**

## QUESTION 8

Given: $f(x)=x^3-5x^2-8x+12$ and $g(x)=ax+q$. A, B$(2;-16)$ and C$(6;0)$ are the points of intersection of $f$ and $g$.

8.1 Determine the coordinates of the turning points of $f$. (4)

8.2 Determine the other two $x$-intercepts of $f$. (3)

8.3 Sketch the graph of $f$, clearly indicating turning points and intercepts with the axes. (4)

8.4 Determine the values of $a$ and $q$. (2)

8.5 Determine whether the graph is concave up or concave down at point B. (3)

8.6 For which values of $x$, is $f(x)\geq g(x)$? (4)

**[20]**

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## QUESTION 9

A large cruise ship uses fuel at a cost of $4x^2$ rand per hour, where $x$ is the speed of the ship in *km/h*.
Other operating costs, including labour, amount to R1 000 per hour.
**[Hint: distance = speed x time: $s = vt$]**

9.1 Show that the total cost for a trip of 500 km is given by, $C(x) = 2\,000x + \frac{500\,000}{x}$. (3)

9.2 At what speed should the ship travel on this 500 km trip to keep the total cost as low as possible? (5)
**[8]**

## QUESTION 10

10.1 Events A and B are mutually exclusive. It is further given that:
- $3P(B) = P(A)$
- $P(A \text{ or } B) = 0{,}64$

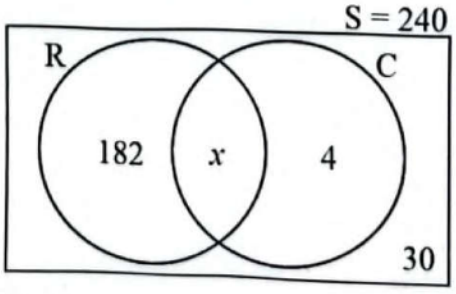
Calculate P(B). (3)

10.2 The probability that it will not rain on a given day is 37%. A child has a 12% chance of falling in dry weather and is three times as likely to fall in wet weather.

10.2.1 Draw a tree diagram to represent ALL the possible ways in which the weather could affect whether a child falls or not. Show the probabilities associated with EACH branch, as well as the outcomes. (4)

10.2.2 What is the probability that a child will not fall on any given day? (2)

10.3 A group of 240 learners were asked whether they play Rugby (R) or Cricket (C) as a school sport. 206 of the learners indicated that they play rugby, 28 said they play cricket, 30 said they play neither and $x$ said they play both. The information is represented in the Venn diagram below.




10.3.1 Determine the value of $x$. (2)

10.3.2 Would you regard playing rugby and cricket as independent events? Support your answer with calculations. (Round answers correct to 2 decimal places.) (3)
**[14]**

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**TOTAL: 150**

# INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \; ; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In $\Delta ABC$:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\,\Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n} \qquad P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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